Principles of Communications

Chapter 3: Analog Modulation
(continued)

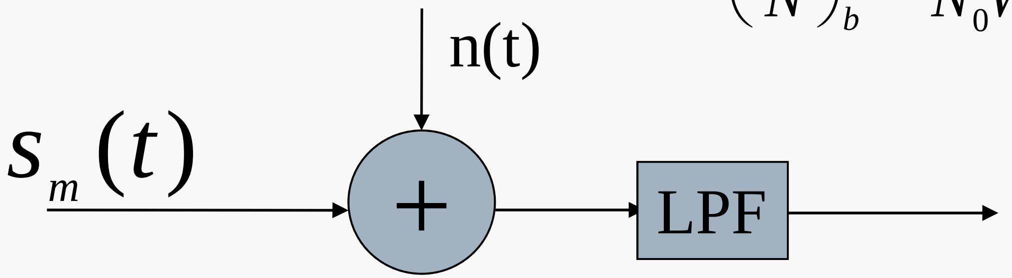
Textbook: Ch 3, Ch 4.1-4.4, Ch 6.1-6.2

3.2 Effect of Noise on AM Systems

- Baseband system (a basis for comparison of various modulation systems):
  - No carrier demodulation
  - The receiver is an ideal LPF with bandwidth W
  - Noise power at the output of the receiver

$$P_{n_0} = \int_{-W}^{W} \frac{N_0}{2} df = N_0 W$$

  - Baseband SNR is given by $\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}$



Example:

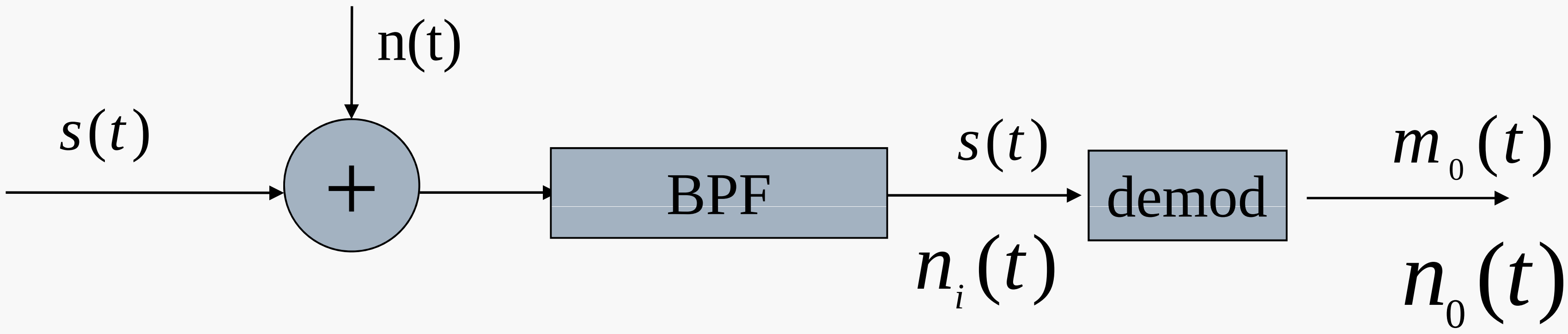
- Find the SNR in a baseband system with a bandwidth of 5 kHz and with $N_0/2 = 10^{-14}$ W/Hz. The transmitter power is 1kW and the channel attenuation is 10^{-12}

- Solution: $P_R = 10^{-12} \times 10^{3} = 10^{-9}\,\text{Watts}$

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{10^{-9}}{10^{-14} \times 5000} = 20$$

$$= 10\log_{10} 20 = 13\text{dB}$$

Effect of Noise on DSBSC



- modulated signal $s(t) = A_c m(t) \cos \omega_c t$
- Input to the demodulator

$$r(t) = s(t) + n_i(t)$$
$$= A_c m(t) \cos w_c t + n_c(t) \cos w_c t - n_s(t) \sin w_c t$$

Here $n_i(t)$ is a Gaussian narrow-band noise

$$S_{n_i}(f) = \begin{cases} N_0/2 & |f - f_c| \le W \\ 0 & \text{otherwise} \end{cases}$$

- In the demodulator, the received signal is first multiplied by a locally generated sinusoid signal

$$
\begin{aligned}
r(t)\cos(w_c t+\phi) &= A_c m(t)\cos w_c t\cos(w_c t+\phi) \\
&\quad + n_c(t)\cos w_c t\cos(w_c t+\phi) - n_s(t)\sin w_c t\cos(w_c t+\phi) \\
&= \frac{1}{2}A_c m(t)\cos\phi + \frac{1}{2}A_c m(t)\cos(2w_c t+\phi) \\
&\quad + \frac{1}{2}\left[n_c(t)\cos\phi + n_s(t)\sin\phi\right] \\
&\quad + \frac{1}{2}\left[n_c(t)\cos(2w_c t+\phi) - n_s(t)\sin(2w_c t+\phi)\right]
\end{aligned}
$$

- Assume coherent detector, we have $\phi = 0$

- Then the signal is passed through a LPF with bandwidth W

$$y(t) = \frac{1}{2}[A_c m(t) + n_c(t)]$$ where $$S_{n_c}(f) = S_{n_i}(f - f_c) + S_{n_i}(f + f_c)$$ $$for \quad |f| \le W$$

- The output SNR can thus be defined as

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{\frac{1}{4}A_c^2 P_m}{\frac{1}{4}P_{n_c}} = \frac{A_c^2 P_m}{2WN_0}$$

- Since the received power of DSBSC in baseband is $P_R = \frac{A_c^2 P_m}{2}$
- The output SNR can be rewritten as

$$\left(\frac{S}{N}\right)_{oDSB} = \frac{P_R}{WN_0} = \left(\frac{S}{N}\right)_b$$

DBSSC does not provide any SNR improvement over a simple baseband systems

Effect of Noise on SSB

- Modulated signal: $s(t) = A_c m(t)\cos w_c t \pm A_c \hat{m}(t)\sin w_c t$
- Input to the demodulator

$r(t) = s(t) + n_i(t)$ where $S_n(f) = \begin{cases} N_0/2 & |f - f_c| \le W/2 \\ 0 & \text{otherwise} \end{cases}$

$$= \left[A_c m(t) + n_c(t)\right]\cos w_c t + \left[\pm A_c \hat{m}(t) - n_s(t)\right]\sin w_c t$$

- Output of LPF: $y(t) = \frac{A_c}{2} m(t) + \frac{1}{2} n_c(t)$
- Therefore, the output SNR is

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{\frac{1}{4}A_c^2 P_m}{\frac{1}{4}P_{n_c}} = \frac{A_c^2 P_m}{WN_0}$$

- But in this case,

$$P_R = A_c^2 P_m$$

- Thus,

$$\left(\frac{S}{N}\right)_{oSSB} = \frac{P_R}{WN_0} = \left(\frac{S}{N}\right)_b$$

⇒ SNR in an SSB system is equivalent to that of a DSBSC system

Effect of Noise on Conventional AM

- Modulated signal

$$s(t) = A_c\left[1 + am(t)\right]\cos w_c t$$

- Input to the demodulator (coherent detector)

$$r(t) = s(t) + n_i(t)$$

$$= \left[A_c + A_c am(t) + n_c(t)\right]\cos w_c t - n_s(t)\sin w_c t$$

- After mixing and low pass filter

$$y_1(t) = \frac{1}{2}\left[A_c + A_c am(t) + n_c(t)\right]$$

- Removing DC component

$$y(t) = \frac{1}{2}\left[A_c am(t) + n_c(t)\right]$$

- In this case, the received signal power $P_R = \frac{1}{2} A_c^2 \left[1 + a^2 P_m \right]$
- Now, we can derive the output SNR as

$$\left(\frac{S}{N}\right)_{oAM} = \frac{\frac{1}{4} A_c^2 a^2 P_m}{\frac{1}{4} n_c} = \frac{A_c^2 a^2 P_m}{2WN_0}$$

$$= \frac{a^2 P_m}{1 + a^2 P_m} \cdot \frac{\frac{A_c^2}{2}\left[1 + a^2 P_m\right]}{N_0 W} = \eta \left(\frac{S}{N}\right)_b$$

Modulation efficiency (η)

⇒ SNR in conventional AM is always smaller than that in baseband.

Performance of Envelope-Detector

- Input to the envelope-detector

$$r(t) = \left[A_c + A_c a m(t) + n_c(t)\right]\cos w_c t - n_s(t)\sin w_c t$$

- Envelope of r(t)

$$V_r(t) = \sqrt{\left[A_c + A_c a m(t) + n_c(t)\right]^2 + n_s^2(t)}$$

- If signal component is much stronger than noise

$$V_r(t) \approx A_c + A_c a m(t) + n_c(t)$$

- After removing DC component, we obtain

$$y(t) = A_c a m(t) + n_c(t)$$

⇨ At high SNR, performance of coherent detector and envelop detector is the same

Performance of Envelope-Detector

- If noise power is much stronger than the signal power

$$V_r(t)=\sqrt{A_c^2\left(1+am(t)\right)^2+n_c^2(t)+n_s^2(t)+2A_cn_c(t)\left(1+am(t)\right)}$$

Ignore 1st term

$$\approx\sqrt{\left(n_c^2(t)+n_s^2(t)\right)\left[1+\frac{2A_cn_c(t)}{n_c^2(t)+n_s^2(t)}\left(1+am(t)\right)\right]}$$

$$V(t)_n=\sqrt{n_c^2(t)+n_s^2(t)}$$

$$\sqrt{1+\varepsilon}\approx1+\frac{\varepsilon}{2}\qquad\approx V_n(t)\left[1+\frac{A_cn_c(t)}{V_n^2(t)}\left(1+am(t)\right)\right]$$

$$=V_n(t)+\frac{A_cn_c(t)}{V_n(t)}\left(1+am(t)\right)$$

⇨ The system is operating below the threshold, no meaningful SNR can be defined.

Exercise

- Consider that the message is a WSS r.p M(t) with autocorrelation function $R_M(\tau) = 16\text{sinc}^2(10000\tau)$. It is given that $|m(t)|_{\max} = 6$.
- We want to transmit this message to a destination via a channel with a 50dB attenuation and additive white noise with PSD $S_n(f) = N_0 / 2 = 10^{-12}\,\text{W/Hz}$. We also want to achieve an SNR at the modulator output of at least 50dB.
- What is the required transmitted power and the channel bandwidth if we employ the following modulation schemes?
  - DSB-SC
  - SSB
  - AM with modulation index = 0.8

3.3 Angle Modulation

- Angle modulation is either phase or frequency of the carrier is varied according to the message signal
- The general form of an angle modulated wave is

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

where f_c = carrier freq, $\theta(t)$ is the time-varying phase and varied by the message m(t)

- The instantaneous frequency of s(t) is

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

Representation of FM and PM signals

- For phase modulation (PM), we have

$$\theta(t) = k_p m(t)$$

where k_p = phase deviation constant

- For frequency modulation (FM), we have

$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

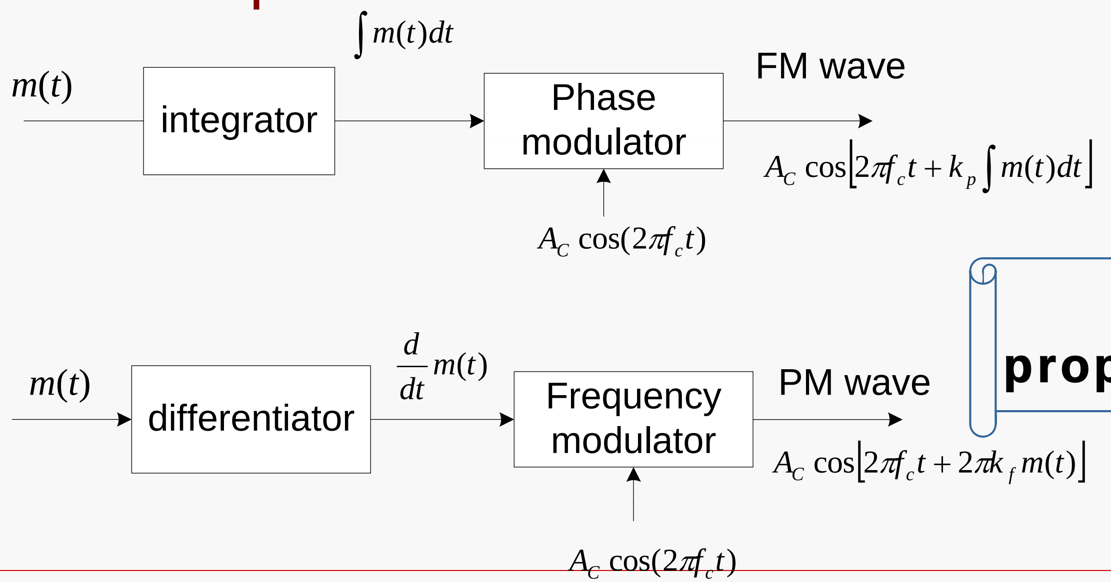
where k_f = frequency deviation constant

- The phase of FM is

$$\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$$

Distinguishing Features of PM and FM

- No perfect regularity in spacing of zero crossing
  - Zero crossings refer to the time instants at which a waveform changes between negative and positive values
- Constant envelop, i.e. amplitude of s(t) is constant
- Relationship between PM and FM



Discuss the properties of FM only

Example: Sinusoidal Modulation

Sinusoid modulating wave m(t)

FM wave

$$\frac{d}{dt}m(t)$$

PM wave

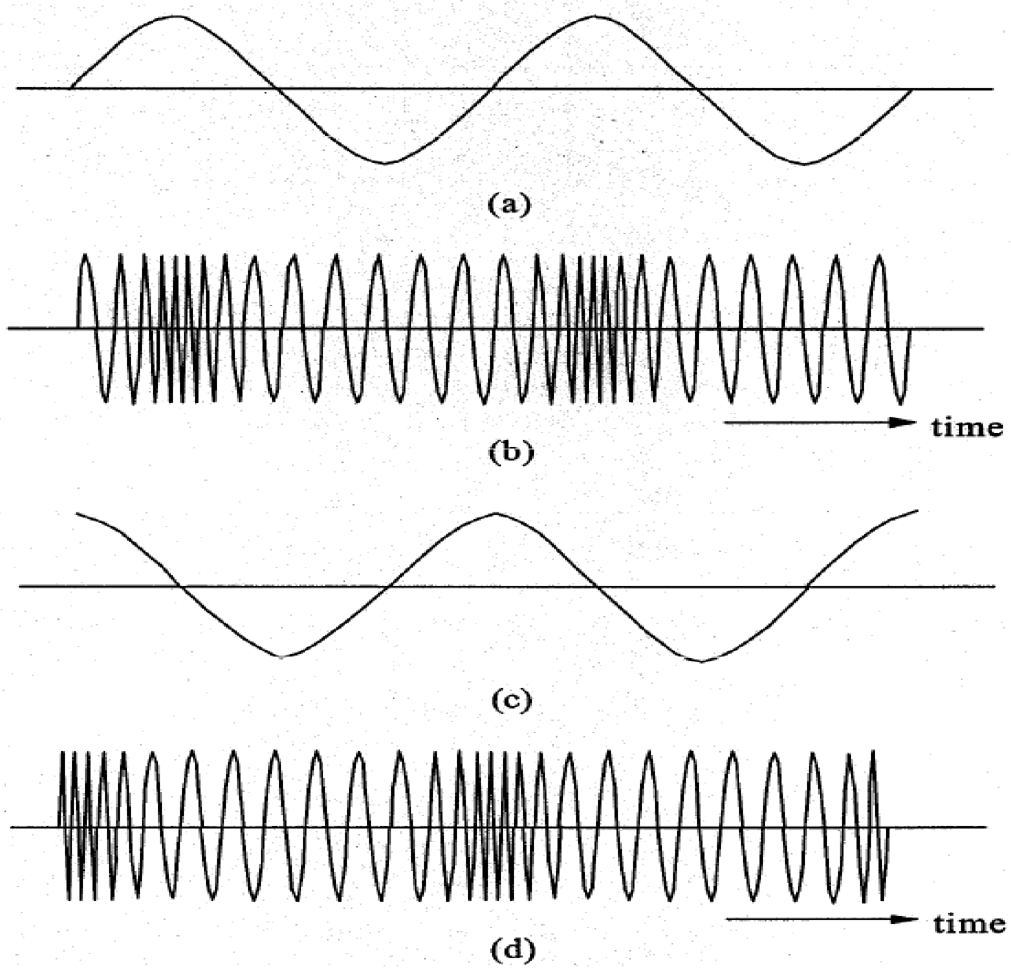


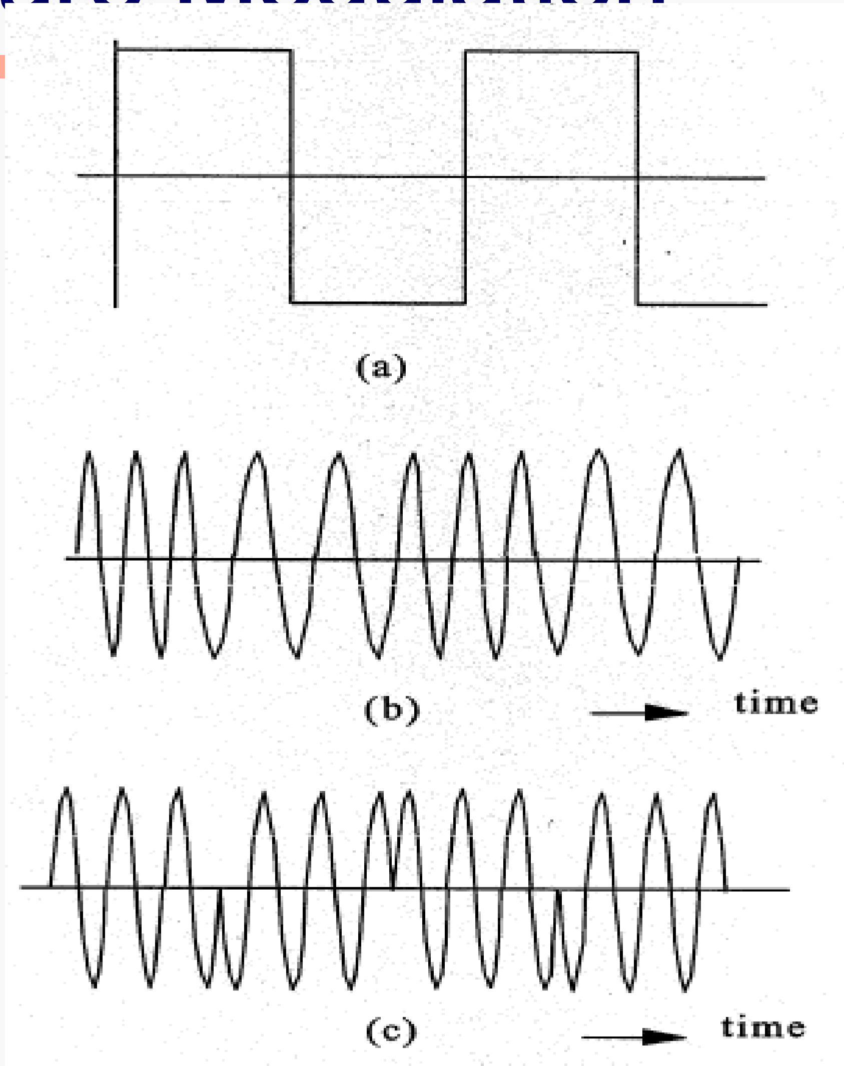
Figure 8.2 - (a) Sinusoidal modulation wave m(t). (b) FM wave. (c) Derivative of *m(t)* with respect to time. (d) PM wave.

Example: Square Modulation

Square modulating wave m(t)

FM wave

PM wave



FM by a Sinusoidal Signal

- Consider a sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

- Instantaneous frequency of resulting FM wave is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t)$$

where $\Delta f = k_f A_m$ is called the frequency deviation, proportional to the amplitude of $m(t)$, and independent of f_m.

- Hence, the carrier phase is

$$\theta(t) = 2\pi \int_0^t \left(f_i(\tau) - f_c \right) d\tau = \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$= \beta \sin(2\pi f_m t)$$

Where $\beta = \Delta f / f_m$ is called the modulation index

Example

- Problem: a sinusoidal modulating wave of amplitude 5V and frequency 1kHz is applied to a frequency modulator. The frequency sensitivity is 40Hz/V. The carrier frequency is 100kHz. Calculate (a) the frequency deviation, and (b) the modulation index

- Solution:
  - Frequency deviation $\Delta f = k_f A_m = 40 \times 5 = 200 Hz$
  - Modulation index $\beta = \frac{\Delta f}{f_m} = \frac{200}{1000} = 0.2$

Spectrum Analysis of Sinusoidal FM Wave

- The FM wave for sinusoidal modulation is

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$
$$= \underbrace{A_c \cos\left[\beta \sin(2\pi f_m t)\right]}_{\text{In-phase component}} \cos(2\pi f_c t) - \underbrace{A_c \sin\left[\beta \sin(2\pi f_m t)\right]}_{\text{Quadrature-phase component}} \sin(2\pi f_c t)$$

In-phase component

$$s_I(t) = A_c \cos\left[\beta \sin(2\pi f_m t)\right]$$

Quadrature-phase component

$$s_Q(t) = A_c \sin\left[\beta \sin(2\pi f_m t)\right]$$

- Hence, the complex envelop of FM wave is

$$\tilde{s}(t) = s_I(t) + j s_Q(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

- $\tilde{s}(t)$ retains complete information about s(t)

$$s(t) = \mathrm{Re}\left\{A_c e^{j[2\pi f_c t + \beta \sin(2\pi f_m t)]}\right\} = \mathrm{Re}\left[\tilde{s}(t) e^{j2\pi f_c t}\right]$$

- $\tilde{s}(t)$ is periodic, can be expanded in Fourier series as

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

where

$$c_n = f_m \int_{-1/(2f_m)}^{1/(2f_m)} \tilde{s}(t) e^{-j2\pi n f_m t} dt$$

$$= f_m A_c \int_{-1/(2f_m)}^{1/(2f_m)} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

- Let $x = 2\pi f_m t$

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp\left[j\left(\beta \sin x - nx\right)\right] dx$$

- n-th order Bessel function of the first kind $J_n(\beta)$ is defined as

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left[j\left(\beta \sin x - nx\right)\right] dx$$

- Hence,

$$c_n = A_c J_n(\beta)$$

- Substituting c_n into $\tilde{s}(t)$

$c(t) = A_c J_n(\beta)$

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left(j2\pi n f_m t \right)$$

- Hence, FM wave in time domain can be represented by

$$s(t) = A_c \operatorname{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \exp\left[j2\pi(f_c + nf_m)t \right] \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi(f_c + nf_m)t \right]$$

- In frequency-domain, we have

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m) \right]$$

- Property 1: Narrowband FM (for small β≤0.3)
  - Approximations

$$J_0(\beta) \approx 1$$
$$J_1(\beta) \approx \beta/2$$
$$J_n(\beta) \approx 0, n > 1$$

? In what ways do a conventinal AM wave and a narrow band FM wave differ from each other

  - Substituting above into s(t)

$$s(t) \approx A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2} \cos[2\pi(f_c + f_m)t] - \frac{\beta A_c}{2} \cos[2\pi(f_c - f_m)t]$$

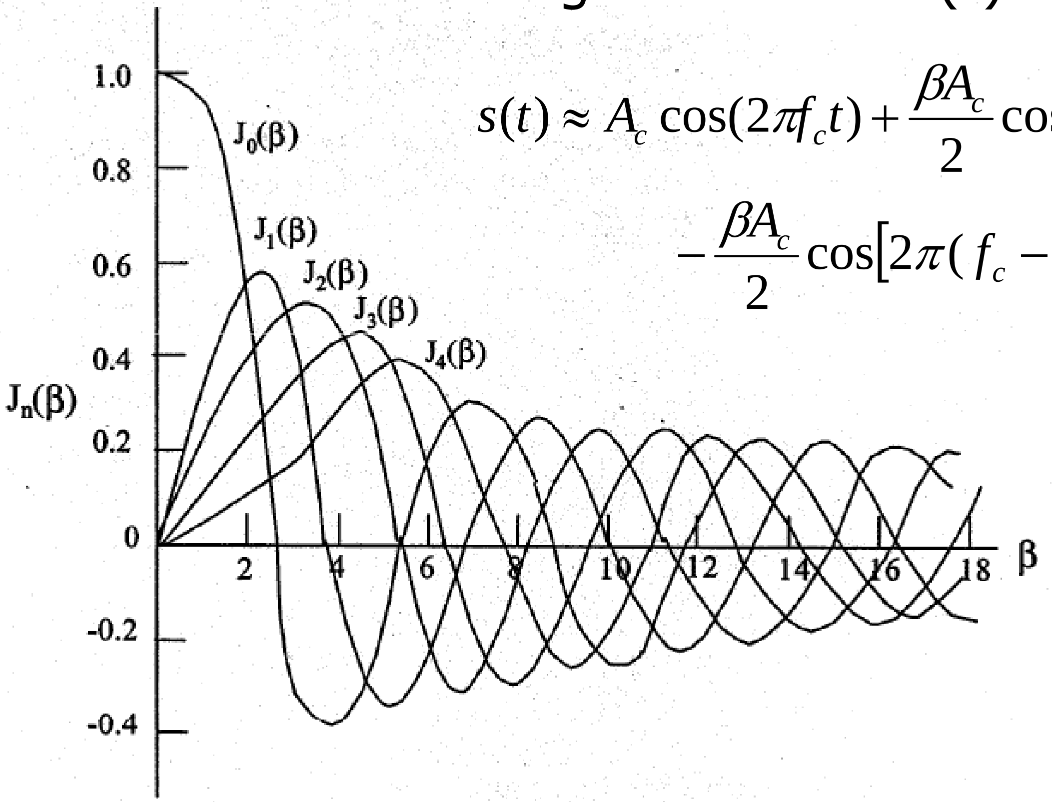


Figure 8.4 - Plots of Bessel functions of the first kind.

$$|J_n(\beta)| \to 0 \text{ as } \beta \to \infty$$

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}$$

- Property 2: Wideband FM (for large β>1)
  - In theory, s(t) contains a carrier and an infinite number of side-frequency components, with no approximations made
- Property 3: Constant average power
  - The envelop of FM wave is constant, so the average power is also constant, $P = A_c^2 / 2$
  - The average power is also given by

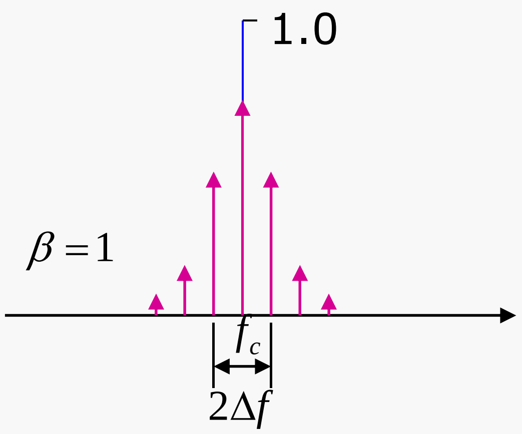
$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)]$$

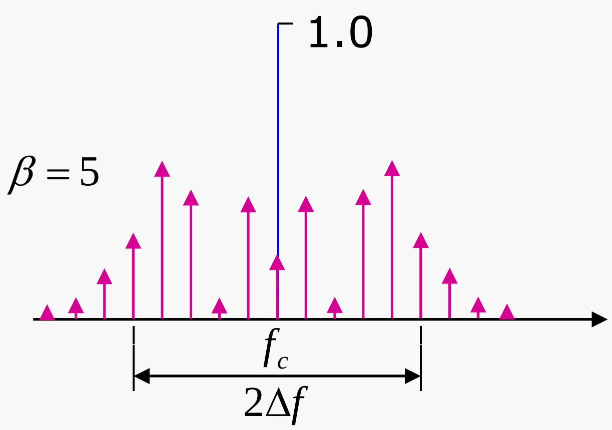
$$P = \frac{A_c^2}{2} \sum_n J_n^2(\beta) = \frac{A_c^2}{2}$$

$$\Rightarrow \quad \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Example

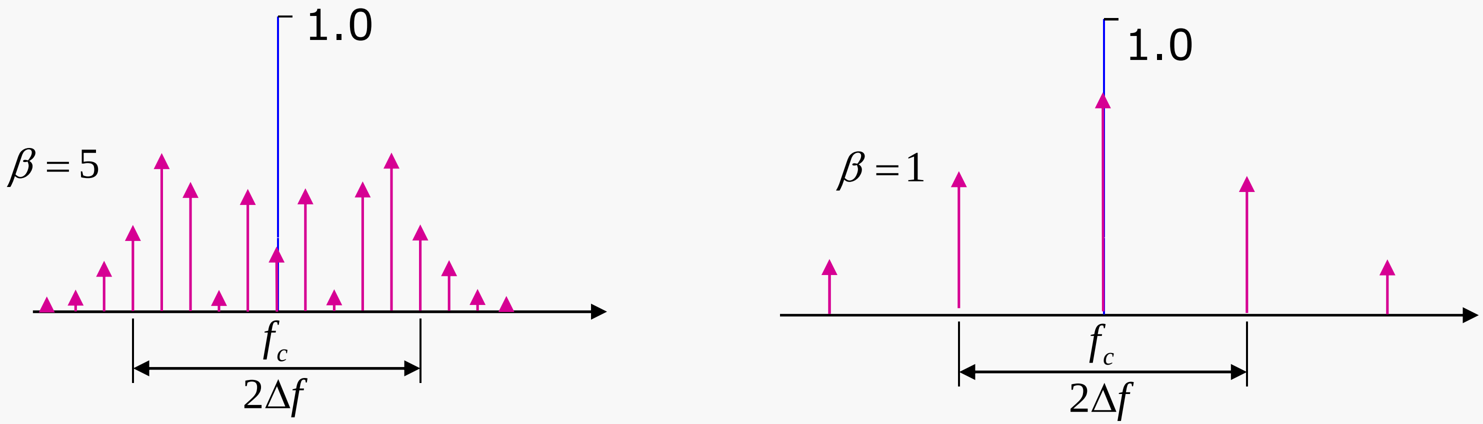
- <u>Goal</u>: to investigate how the amplitude A_m, and frequency f_m, of a sinusoidal modulating wave affect the spectrum of FM wave
- Fixed f_m and varying $A_m \Rightarrow$ frequency deviation $\Delta f = k_f A_m$ and modulation index $\beta = \Delta f / f_m$ are varied





- Increasing A_m increases the number of harmonics in the bandwidth

- Fixed A_m and varying $f_m \Rightarrow \Delta f$ is fixed, but β is varied



  - Increasing f_m decreases the number of harmonics but at the same time increases the spacing between the harmonics.

Effective Bandwidth of FW Waves

- Theoretically, FM bandwidth = infinite
- In practice, for a single tone FM wave, when β is large, B is only slightly greater than the total frequency excursion $2\Delta f$. when β is small, the spectrum is effectively limited to $[f_c - f_m, f_c + f_m]$
- Carson's Rule approximation for single-tone modulating wave of frequency f_m

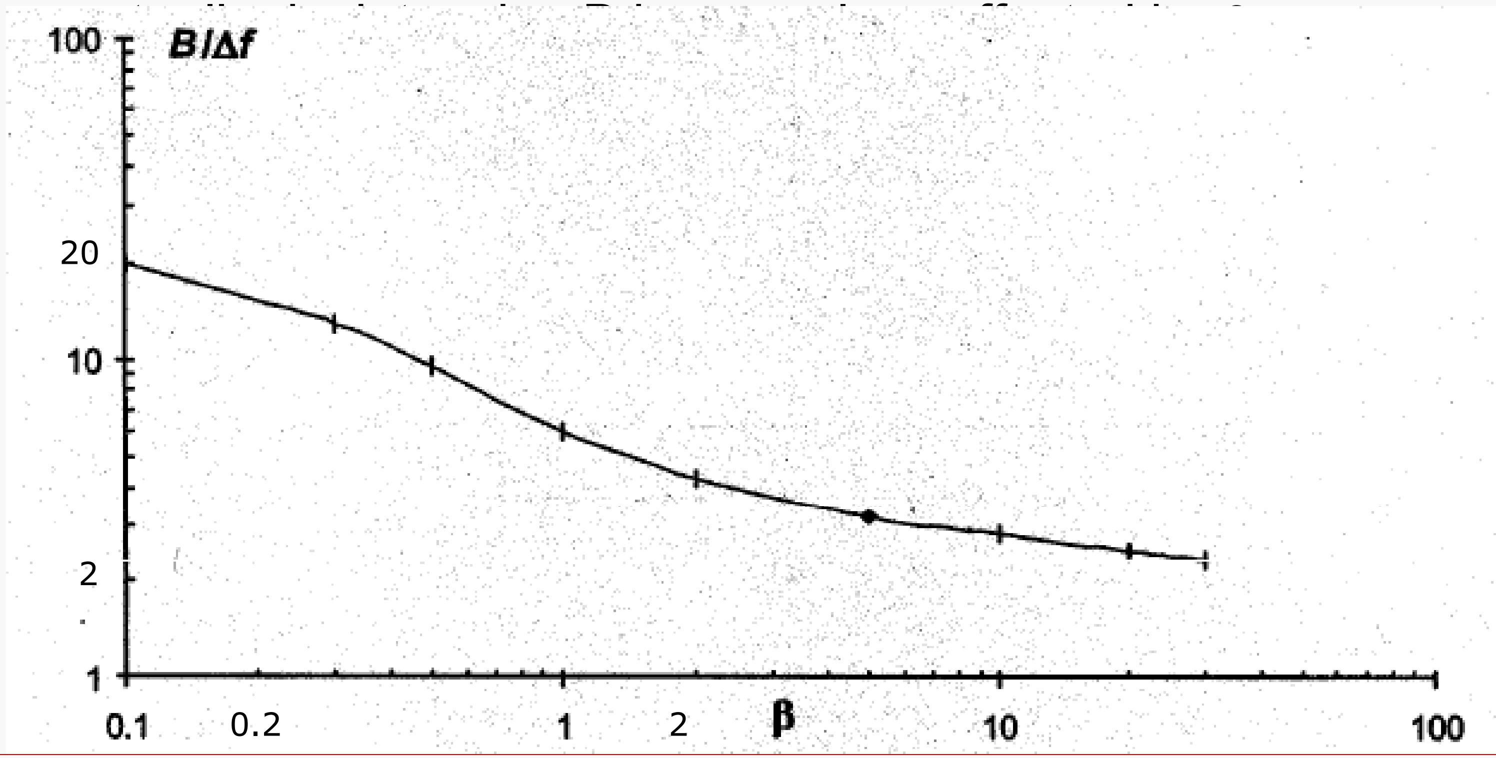
$$B \approx 2\Delta f + 2f_m = 2(1+\beta)f_m$$

- 99% bandwidth approximation
  - The separation between the two frequencies beyond which none of the side-frequencies is greater than 1% of the unmodulated carrier amplitude
  - i.e $B \approx 2n_{\max} f_m$ where $n_{\max}$ is the max n that satisfies

$$|J_n(\beta)| > 0.01$$

β	0.1	0.3	0.5	1.0	2.0	5.0	10	20	30
$2n_{max}$	2	4	4	6	8	16	28	50	70

- A universal curve for evaluating the 99% bandwidth
  - As β increases, the bandwidth occupied by the significant side-frequencies drops toward that over which the carrier frequency



FM by an Arbitrary Message

- Consider an *arbitrary* m(t) with highest freq component W
- Define deviation ratio D = Δf / W, where $\Delta f = k_f \max|m(t)|$

  D $\Leftrightarrow$ β and W $\Leftrightarrow$ f_m
- Carson's rule applies as

$$B \approx 2\Delta f + 2W = 2W(1+D)$$

- Carson's rule somewhat underestimate the FM bandwidth requirement, while universal curve yields a somewhat conservative result
- Assess FM bandwidth between the bounds given by Carson's rule and the universal curve

Example

- In north America, the maximum value of frequency deviation Δf is fixed at 75kHz for commercial FM broadcasting by ratio.
- If we take the modulation frequency $W = 15\text{kHz}$, which is typically the maximum audio frequency of interest in FM transmission, the corresponding value of the deviation ratio is $D = 75/15 = 5$
- Using *Carson's rule*, the approximate value of the transmission bandwidth of the FM wave is

$$B = 2\,(75+15) = 180\text{kHz}$$

- Using *universal curve*,

$$B = 3.2\,\Delta f = 3.2 \times 75 = 240\text{kHz}$$

Exercise

- Assuming that $m(t) = 10\text{sinc}\left(10^4 t\right)$, determine the transmission bandwidth of an FM modulated signal with $k_f = 4000$

Generation of FM waves

- Direct approach
  - Design an oscillator whose frequency changes with the input voltage => voltage-controlled oscillator (VCO)
- Indirect approach
  - First generate a narrowband FM signal and then change it to a wideband signal
  - Due to the similarity of conventional AM signals, the generation of a narrowband FM signal is straightforward.

Generation of Narrow-band FM

- Consider a narrow band FM wave

$$s_1(t) = A_1 \cos\left[2\pi f_1 t + \phi_1(t)\right]$$

where $\phi_1(t) = 2\pi k_1 \int_0^t m(\tau) d\tau$

f_1 = carrier frequency

k_1 = frequency sensitivity

- Given $\phi_1(t) << 1$ with $\beta \leq 0.3$, we may use

$$\begin{cases} \cos\left[\phi_1(t)\right] \approx 1 \\ \sin\left[\phi_1(t)\right] \approx \phi_1(t) \end{cases}$$

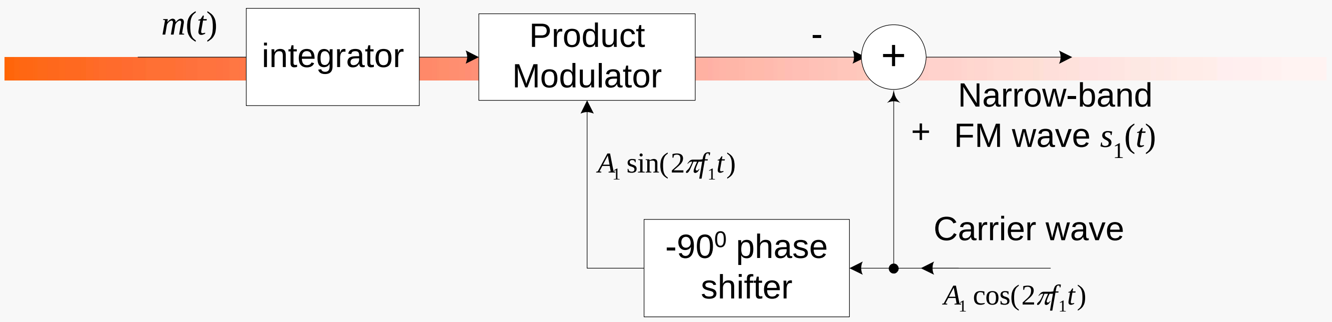
- Correspondingly, we may approximate $s_1(t)$ as

$$s_1(t) = A_1 \cos(2\pi f_1 t) - A_1 \sin(2\pi f_1 t)\phi_1(t)$$

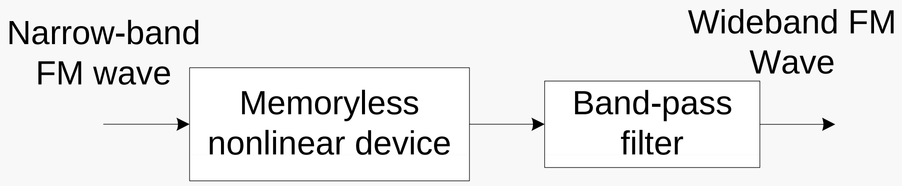
$$= A_1 \cos(2\pi f_1 t) - 2\pi k_1 A_1 \sin(2\pi f_1 t)\int_0^t m(\tau) d\tau$$

Narrow-band FW wave

- Narrow-band frequency modulator



- Next, pass $s_1(t)$ through a frequency multiplier, which consists of a non-linear device and a bandpass filter.



  – The input-output relationship of the non-linear device is modeled as

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t) + \ldots + a_n s_1^n(t)$$

  – The BPF is used to Pass the FM wave centred at nf_1 and with deviation $n\Delta f_1$ and suppress all other FM spectra

Example: frequency multiplier with n = 2

- Problem: Consider a square-law device based frequency multiplier

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t)$$

with

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau) d\tau \right)$$

- Specify the midband freq. and bandwidth of BPF used in the freq. multiplier for the resulting freq. deviation to be twice that at the input of the nonlinear device
- Solution:

$$s_2(t) = a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau) d\tau \right) + a_2 A_1^2 \cos^2\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau) d\tau \right)$$

$$= \underbrace{a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau) d\tau \right) + \frac{a_2 A_1^2}{2}}_{\text{Removed by BPF with}} + \frac{a_2 A_1^2}{2} \cos\left(4\pi f_1 t + 4\pi k_1 \int_0^t m(\tau) d\tau \right)$$

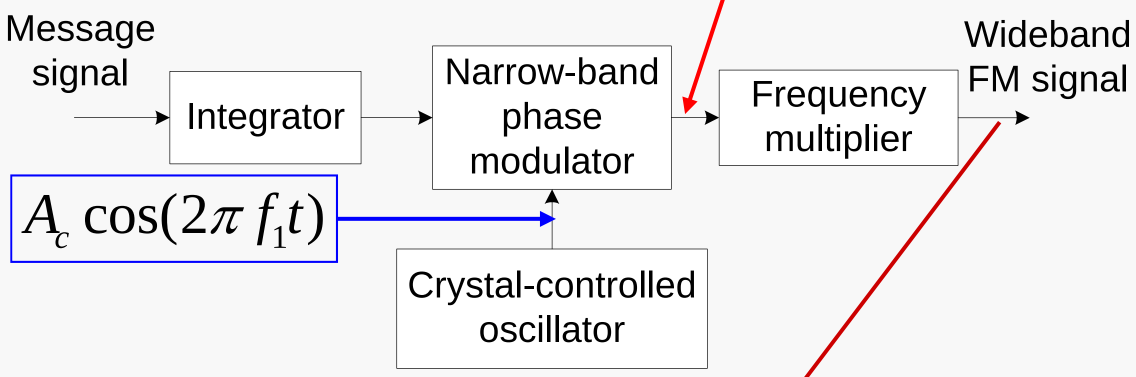
Removed by BPF with

$f_c = 2f_1$

$BW > 2\Delta f = 4\Delta f_1$

- Thus, connecting the narrow-band frequency modulator and the frequency multiplier, we may build the wideband frequency modulator

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau) d\tau \right)$$



$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

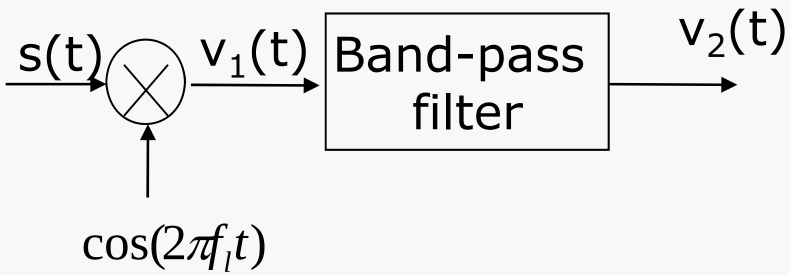
$$f_c = n f_1$$

$$k_f = n k_1$$

$$\Delta f = n \Delta f_1$$

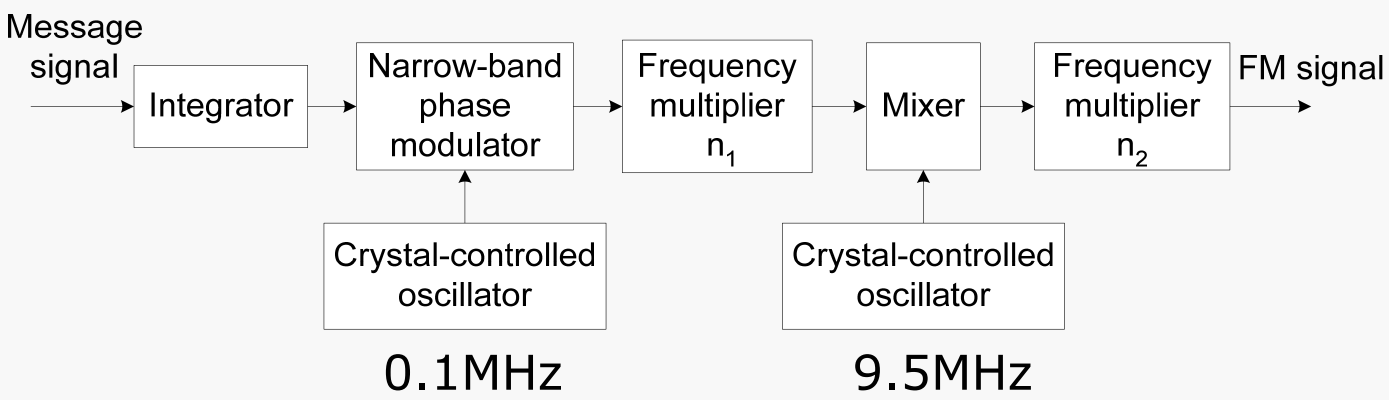
Mixer

- $f_c = nf_1$ may not be the desired carrier frequency. The modulator performs an up/down conversion to shift the modulated signal to the desired center freq.
- This consists of a mixer and a BPF



Exercise: A typical FM transmitter

- Problem: Given the simplified block diagram of a typical FM transmitter used to transmit audio signals containing frequencies in the range 100Hz to 15kHz.
- Desired FM wave: f_c = 100MHz, Δf = 75kHz.
- Set β_1 = 0.2 in the narrowband phase modulation to limit harmonic distortion.
- Specify the two-stage frequency multiplier factors n_1 and n_2



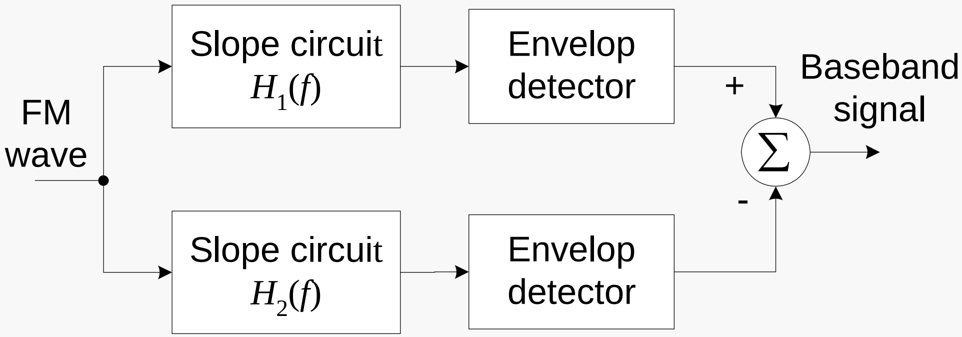
Demodulation of FM
Balanced Frequency Discriminator

- Given FM wave $s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$

$$\frac{d}{dt}s(t) = -A_c\left[2\pi f_c + 2\pi k_f m(t)\right]\sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

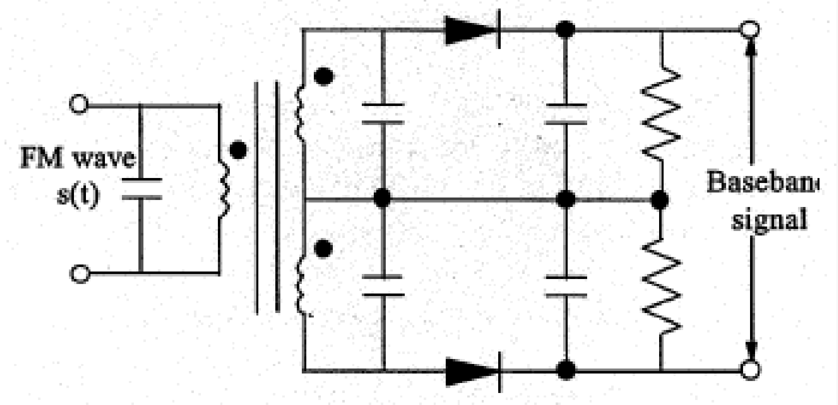
⇒ Hybrid-modulated wave with AM and FM

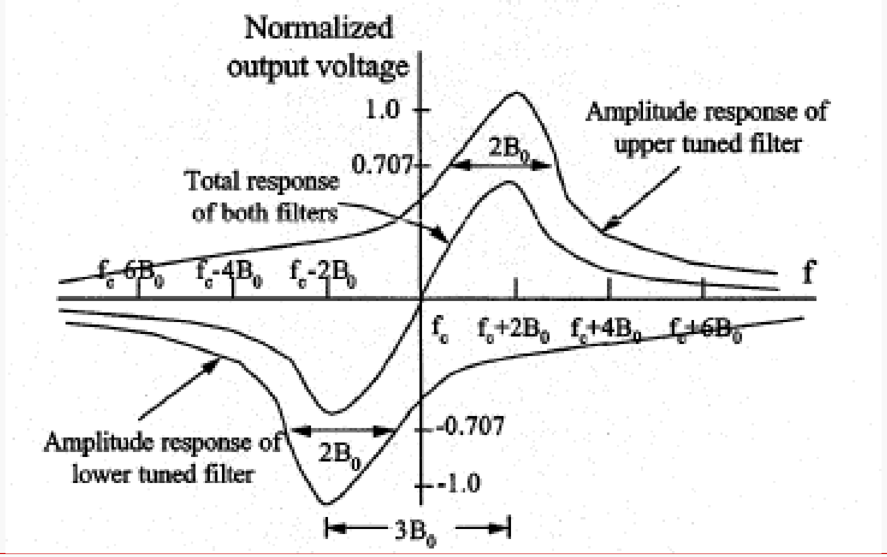
- Differentiator + envelop detector = FM demodulator
- Frequency discriminator: a "freq to amplitude" transform device



$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + B/2\right), & f_c - B/2 \le f \le f_c + B/2 \\ j2\pi a\left(f + f_c - B/2\right), & -f_c - B/2 \le f \le -f_c + B/2 \\ 0, & \text{elsewhere} \end{cases} \qquad H_2(f) = H_1(-f)$$

Circuit diagram and frequency response





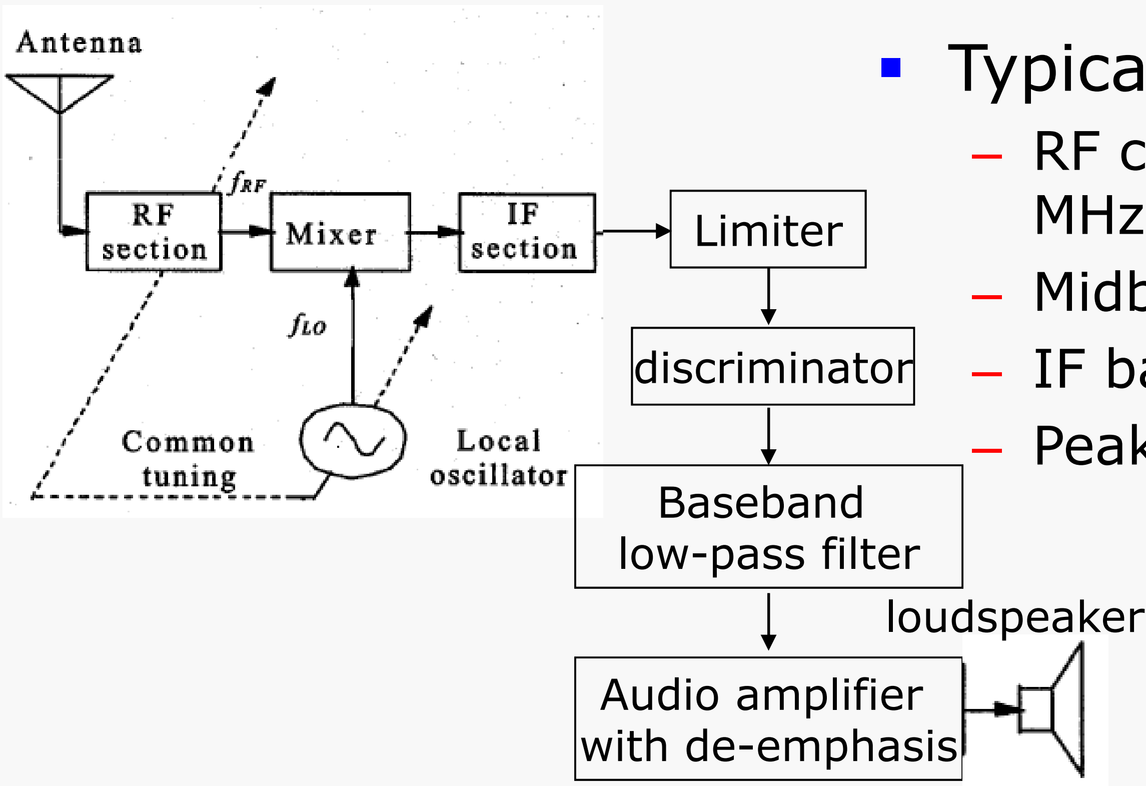
Think …

- Compared with amplitude modulation, angle modulation requires a higher implementation complexity and a higher bandwidth occupancy.

- What is the usefulness of angle modulation systems?

Application: FM Radio broadcasting

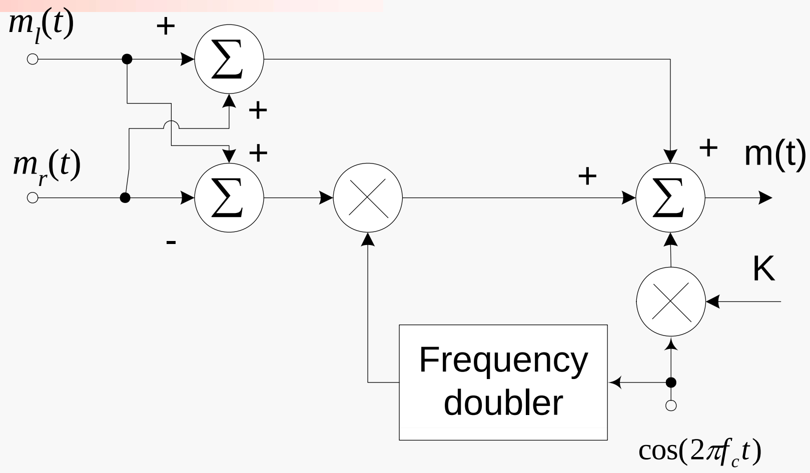
- As with standard AM radio, most FM radio receivers are of super-heterodyne type



- Typical freq parameters
  - RF carrier range = 88~108 MHz
  - Midband of IF = 10.7MHz
  - IF bandwidth = 200kHz
  - Peak freq. deviation = 75KHz

FM Radio Stereo Multiplexing

- Stereo multiplexing is a form of FDM designed to transmit two separate signals via the same carrier.
- Widely used in FM broadcasting to send two different elements of a program (e.g. vocalist and accompanist in an orchestra) so as to give a spatial dimension to its perception by a listener at the receiving end

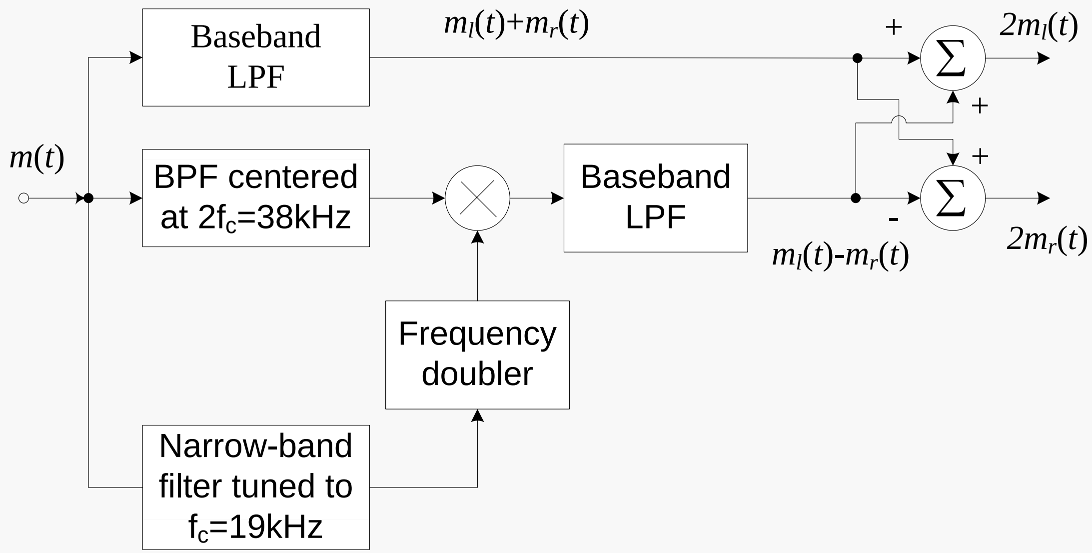


$$m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)]\cos(4\pi f_c t) + K\cos(2\pi f_c t)$$

f_c = 19kHz

- The sum signal is left unprocessed in its baseband form
- The difference signal and a 38-kHz subcarrier produce a DSBSC wave
- The 19-kHz pilot is included as a reference for coherent detection

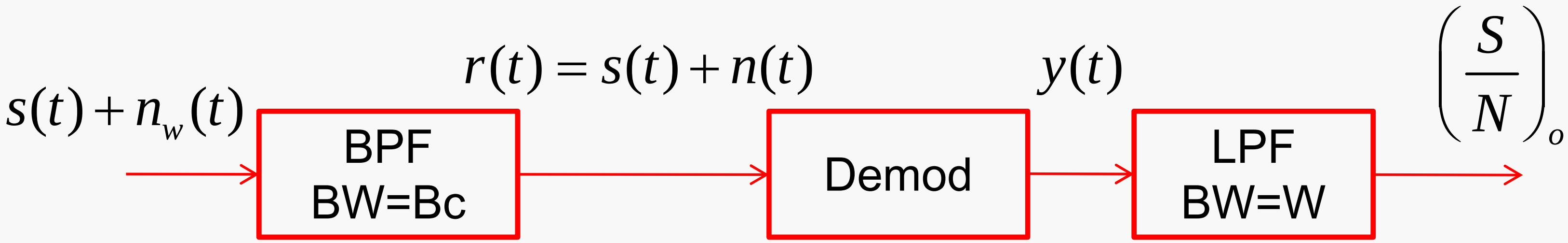
FM-Stereo Receiver



To two loudspeakers

3.4 Effect of Noise on Angle Modulation

- Block diagram of an angle demodulator



- Input to the demodulator is

$$r(t) = s(t) + n(t)$$

$$= A_c \cos\left[\omega_c t + \phi(t)\right] + \underbrace{n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t}$$

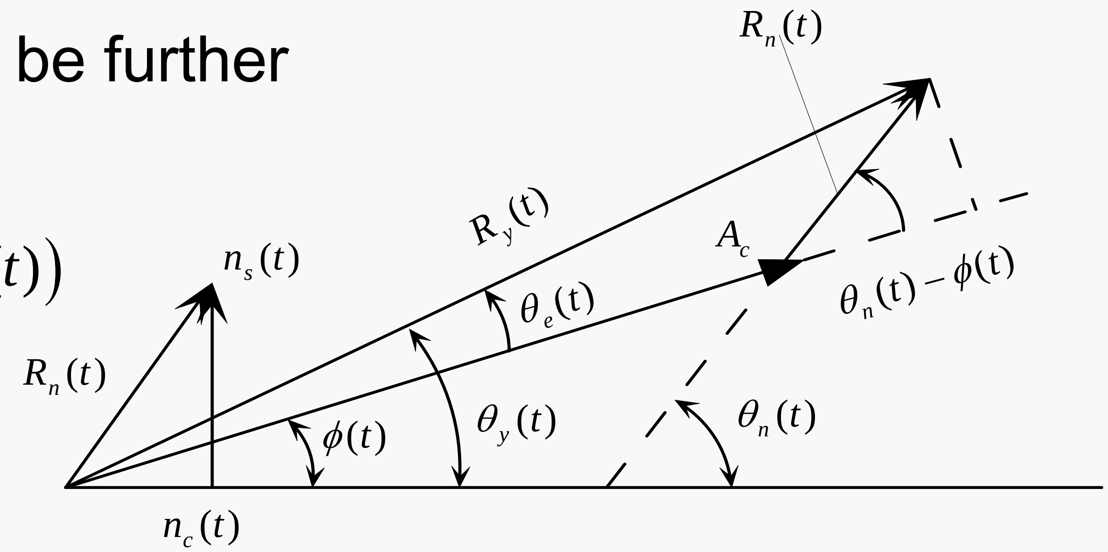
$$= R_n(t)\cos(w_c t + \theta_n(t))$$

- Assume that the signal is much larger than the noise

$$r(t) \approx \left[A_c + R_n(t)\cos\left(\theta_n(t)-\phi(t)\right)\right]\cdot$$
$$\cos\left(w_c t + \phi(t) + tg^{-1}\frac{R_n(t)\sin\left(\theta_n(t)-\phi(t)\right)}{A_c + R_n(t)\cos\left(\theta_n(t)-\phi(t)\right)}\right)$$

- The phase term can be further approximated as

$$\theta_r(t) = \phi(t) + \frac{R_n(t)}{A_c}\sin\left(\theta_n(t)-\phi(t)\right)$$



$A_c^2 \gg E\left[n^2(t)\right]$

- Therefore, the output of the demodulator is

$$y(t) = \frac{d}{2\pi dt}\theta_r(t) = k_f m(t) + \frac{d}{2\pi dt}\frac{R_n(t)}{A_c}\sin\left(\theta_n(t) - \phi(t)\right)$$

$$= \underbrace{k_f m(t)}_{\text{Desired signal}} + \underbrace{\frac{d}{2\pi dt}Y_n(t)}_{\text{Noise}}$$

- The noise component is inversely proportional to the signal amplitude Ac. (This is not the case for AM system)

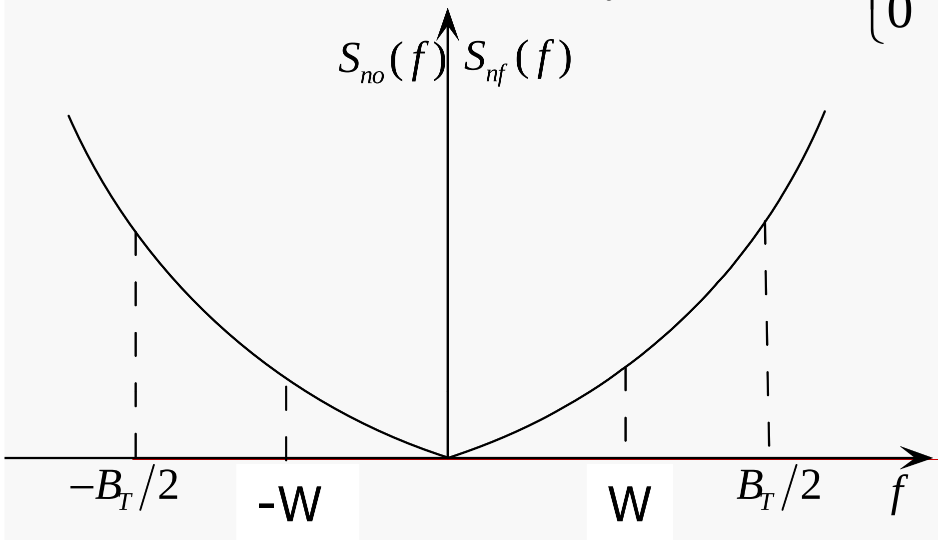
$$Y_n(t) = \frac{R_n(t)}{A_c}\sin\left(\theta_n(t) - \phi(t)\right) = \frac{1}{A_c}\left[n_s(t)\cos\phi(t) - n_c(t)\sin\phi(t)\right]$$

$$= \frac{1}{A_c}\left[n_s(t)\cos\phi - n_c(t)\sin\phi\right]$$ (Since $\phi(t)$ is slowly varying)

- The power spectral density of $\frac{1}{2\pi}\frac{d}{dt}Y_n(t)$ is

$$\frac{4\pi^2 f^2}{4\pi^2} S_{Y_n}(f) = f^2\left[\left(\frac{\cos\phi}{A_c}\right)^2 S_{n_s}(f) + \left(\frac{\sin\phi}{A_c}\right)^2 S_{n_c}(f)\right]$$

$$= \frac{f^2}{A_c^2} S_{n_c}(f) \quad = \begin{cases} \frac{f^2}{A_c^2} N_0 & |f| \le B_C/2 \\ 0 & \text{otherwise} \end{cases}$$



- At the output of LPF, the noise is limited to the freq. range [-W, W]

- Now we can determine the output SNR in FM
- First, the output signal power is

$$P_{s_o} = k_f^2 P_m$$

- The output noise power is

$$P_{n_o} = \int_{-W}^{W} \frac{N_0}{A_c^2} f^2 df = \frac{2N_0 W^3}{3A_c^2}$$

- Then, the output SNR is

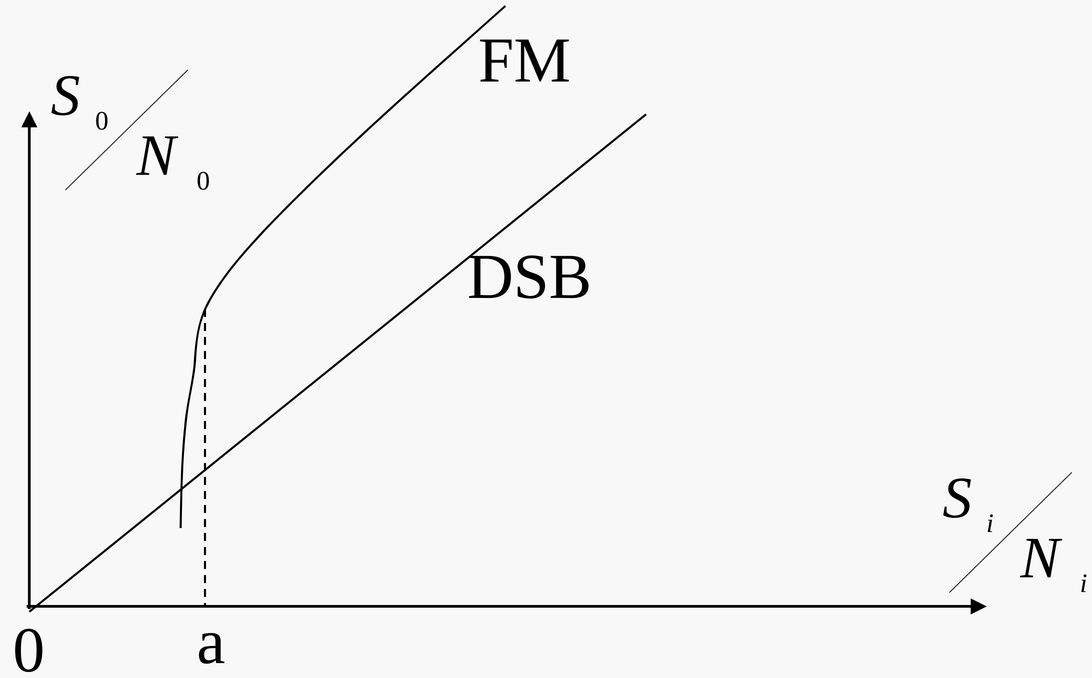
$$\left(\frac{S}{N}\right)_o = \frac{P_{s_0}}{P_{n_o}} = \frac{3k_f^2 A_c^2}{2W^2} \frac{P_m}{N_0 W} = \frac{3\beta_f^2 P_m}{\left(\max|m(t)|\right)^2} \left(\frac{S}{N}\right)_b$$

Observations

- Increasing the modulation index β increases the output SNR, in contrast to AM
- Increasing the bandwidth increases the output SNR. **Therefore, angle modulation provides a way to trade off bandwidth for transmitted power**
- Increasing the transmitted power increases output SNR in both FM and AM systems, but the mechanisms are totally different (explain!)
- Increasing β up to a certain value improves the performance, but cannot continue indefinitely due to the threshold effect

Threshold Effect

- There exists a specific SNR at the input of the demodulator below which the signal is not distinguishable from the noise



Comparison of Analog-Modulation

- Bandwidth efficiency
  - SSB is the most bandwidth efficient, but cannot effectively transmit DC
  - VSB is a good compromise
  - PM/FM are the least favorable systems
- Power efficiency
  - FM provides high noise immunity
  - Conventional AM is the least power efficient
- Ease of implementation (transmitter and receiver)
  - The simplest receiver structure is conventional AM
  - FM receivers are also easy to implement
  - DSB-SC and SSB-SC requires coherent detector and hence is much more complicated.

Applications

- SSB-SC:
  - Voice transmission over microwave and satellite links
- VSB-SC
  - Widely used in TV broadcasting
- FM
  - High-fidelity radio broadcasting
- Conventional AM
  - AM radio broadcasting
- DSB-SC
  - Hardly used in analog signal transmission!