

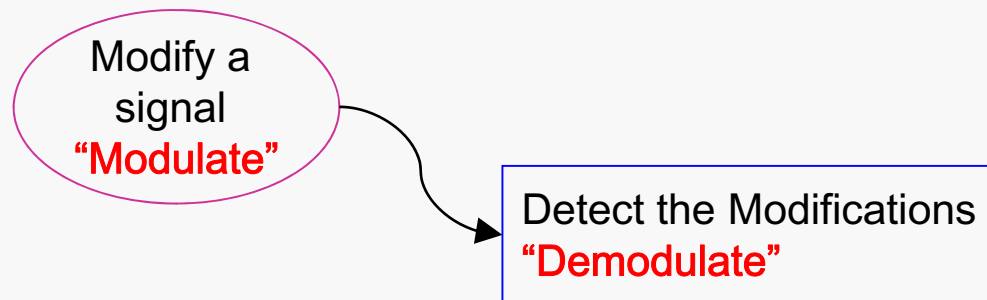
Principles of Communications

Chapter 3: Analog Modulation
Part I: Amplitude Modulation

Textbook: Ch3

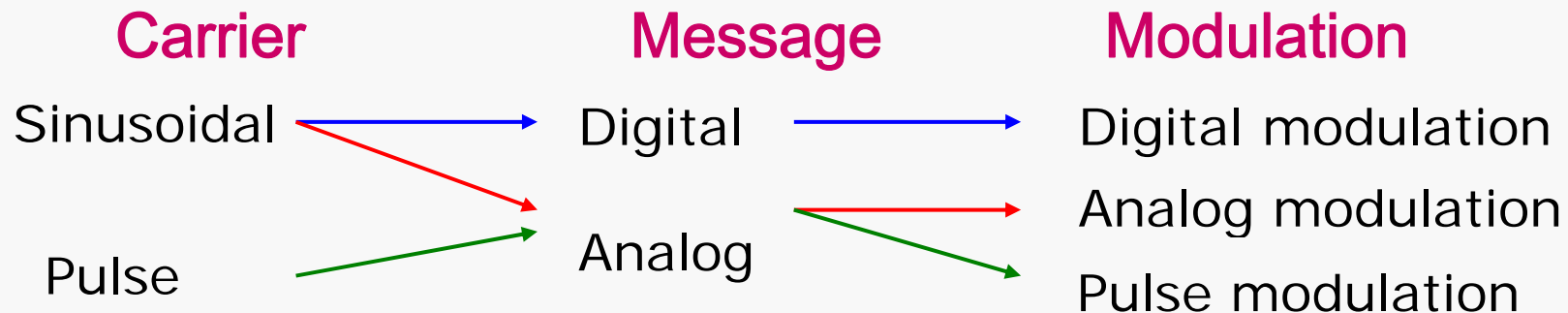
Modulation

- ❑ 3-step signal transmission over a band-pass channel
 - A pure **carrier** (usually sinusoidal) is generated at the transmitter
 - The carrier is modulated with **the information** to be transmitted. Any reliably detectable change in signal characteristics can carry information
 - At the receiver the signal modifications or changes are detected and demodulated



Modulation

- ❑ Modulation objectives
 - Frequency translation from lowpass to passband
 - Frequency-division multiplexing
 - Increasing noise and interference immunity
- ❑ Modulation types



Analog Modulation

- Characteristics that be modified in sin carrier
 - Amplitude → Amplitude modulation
 - Frequency } → Angle modulation
 - Phase }

- In the following we
 - Consider the transmission and reception of analog signals by amplitude modulation
 - Compare their bandwidth requirement and implementation complexity
 - Discuss the performance in the presence of noise

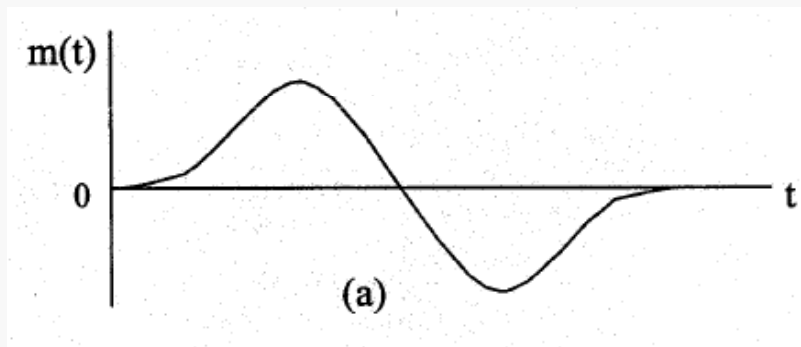
Analog Modulation

- ◆ 3.1. Amplitude modulation
- ◆ 3.2. Effect of noise on AM systems
- ◆ 3.3. Angle modulation
- ◆ 3.4. Effect of noise on angle modulation

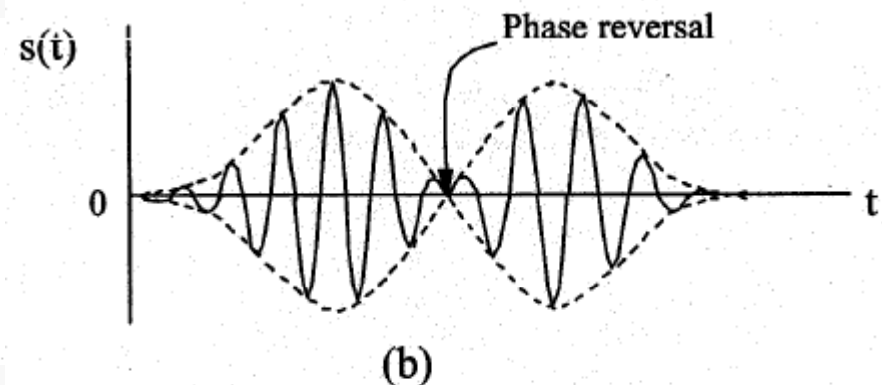
Double-Sideband Suppressed-Carrier AM (DSB-SC)

- ❑ Carrier wave : $c(t) = A_c \cos(\omega_c t + \theta_0)$
- ❑ Baseband signal (modulating wave) $m(t)$
- ❑ Modulated wave

$$s(t) = c(t)m(t) = A_c m(t) \cos(\omega_c t + \theta_0)$$



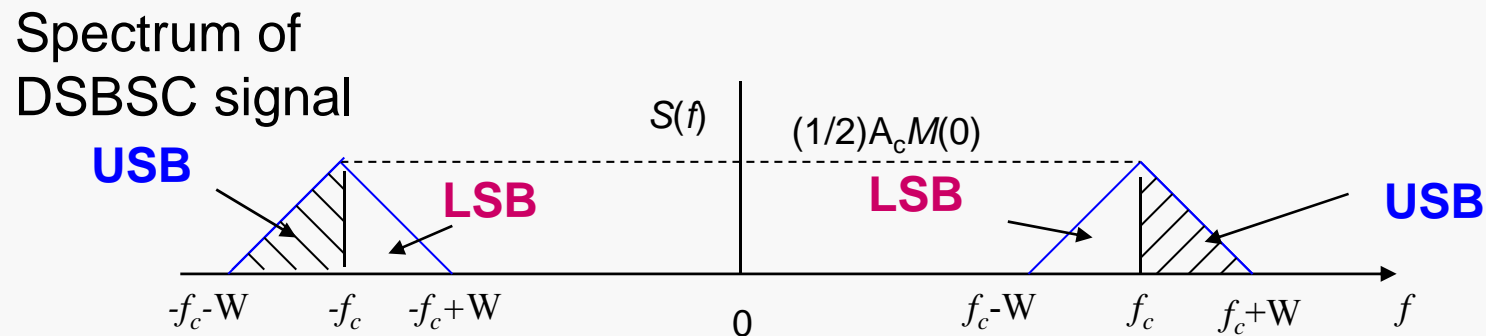
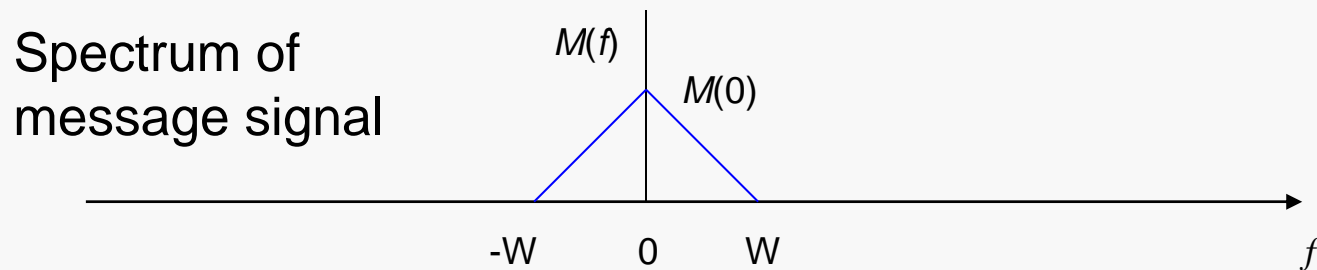
Modulating wave



Modulated wave

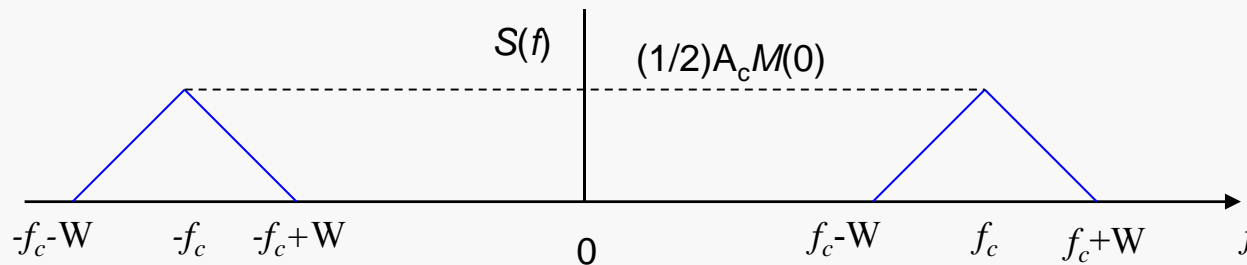
Spectrum of DSB-SC Signals

$$S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)]$$



- Translation of the original message spectrum to $\pm f_c$
- Suppression of the carrier

Bandwidth and Power of DSB-SC

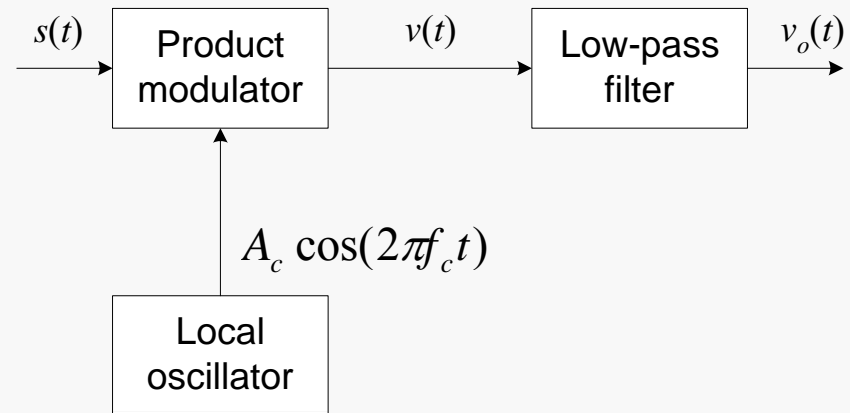


- ❑ Channel bandwidth required to transmit the modulated signal is $B_c = 2W$, 2 times of the message bandwidth
- ❑ Power content

$$\begin{aligned} P_s &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} s^2(t) dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \cos^2(\omega_c t + \theta_0) dt \\ &= \frac{A_c^2}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) [1 + \cos(2\omega_c t + 2\theta_0)] dt = \frac{A_c^2}{2} P_m \end{aligned}$$

Demodulation of DSB-SC Signals

- The local oscillator is assumed to be exactly coherent or synchronized to original $c(t)$ => coherent detection or synchronous detection



- If there is a phase error ϕ , then

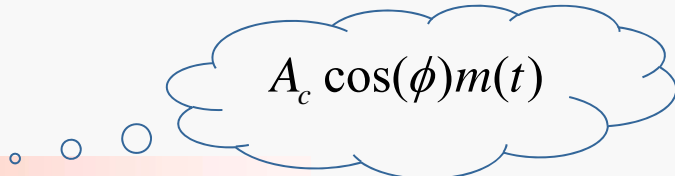
$$v(t) = \cos(2\pi f_c t + \phi) s(t) = A_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$

$$= \underbrace{\frac{1}{2} A_c \cos(\phi) m(t)}_{\text{Scaled version of message signal}} + \underbrace{\frac{1}{2} A_c \cos(4\pi f_c t + \phi) m(t)}_{\text{Unwanted terms}}$$

Scaled version of message signal

Unwanted terms

Phase Error ϕ


$$A_c \cos(\phi)m(t)$$

- ❑ $\cos(\phi) =$ **Attenuation factor**
- ❑ If $\phi = 0 \Rightarrow$ amplitude of demodulated signal is maximized
- ❑ If $\phi = \pm\pi/2 \Rightarrow$ amplitude is zero, called **quadrature null effect**
- ❑ In practice, ϕ varies randomly with time, resulting in undesired effect
- ❑ Need additional circuitry to ensure synchronization
- ❑ **The increased receiver complexity is the price that must be paid for suppressing the carrier wave to save transmit power**

Example: Single-tone DSBSC Modulation

- ❑ Consider a single tone modulating wave $m(t) = A_m \cos(2\pi f_m t)$
- ❑ The DSBSC modulated wave is

$$\begin{aligned} s(t) &= A_c A_m \cos(2\pi f_c t) \cos(2\pi f_m t) \\ &= \frac{1}{2} A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_c A_m \cos[2\pi(f_c - f_m)t] \end{aligned}$$

- ❑ With perfect synchronization, the output of product modulator is

$$\begin{aligned} v(t) &= \cos(2\pi f_c t) s(t) \\ &= \frac{1}{2} A_c A_m \cos(2\pi f_m t) + \frac{1}{4} A_c A_m \cos[2\pi(2f_c - f_m)t] \\ &\quad + \frac{1}{4} A_c A_m \cos[2\pi(2f_c + f_m)t] \end{aligned}$$

Removed by LPF

Conventional Amplitude Modulation

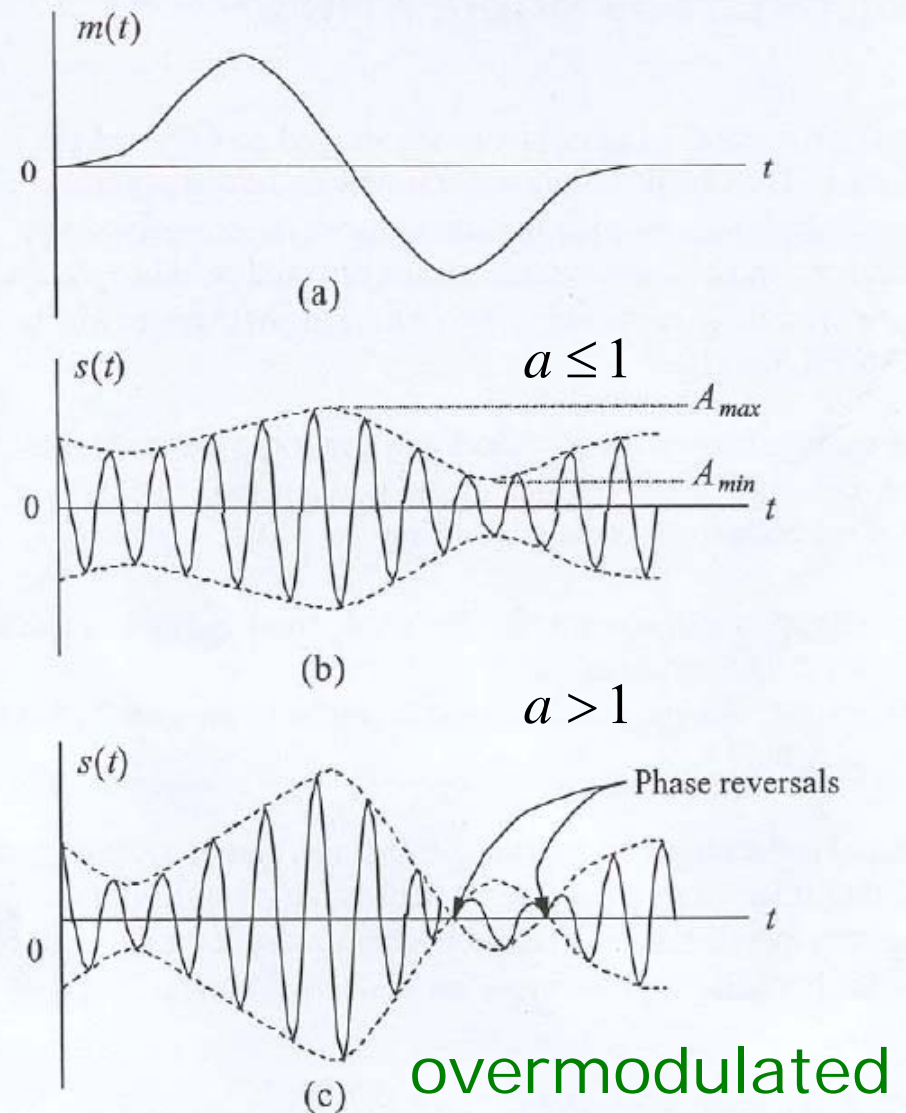
□ Modulated signal :

$$s(t) = A_c [1 + am(t)] \cos(2\pi f_c t)$$

$$= A_c am(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t)$$

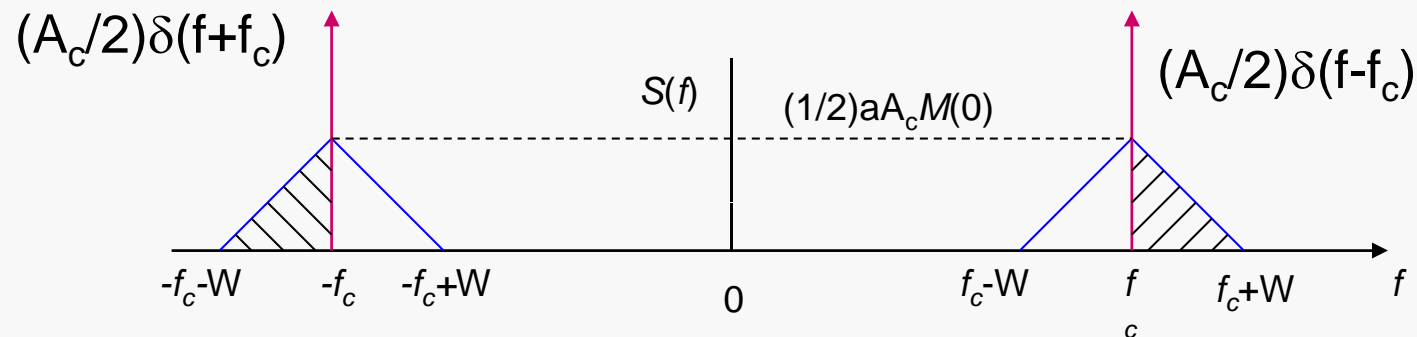
$m(t)$: normalized message

a : modulation index



Spectrum of Conventional AM

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c a}{2} [M(f - f_c) + M(f + f_c)]$$



Power for the Conventional AM

□ Power
$$S = E[s^2(t)] = E\left\{A_c^2 [1 + am(t)]^2 \cos^2 \omega_c t\right\}$$

$$= \frac{A_c^2}{2} + \frac{a^2 A_c^2}{2} E[m^2(t)] = \underbrace{\frac{A_c^2}{2}}_{\text{Power in the carrier component}} + \underbrace{\frac{a^2 A_c^2}{2} P_m}_{\text{Power in sidebands}}$$

□ Modulation efficiency

$$E = \frac{\text{power in sideband}}{\text{total power}} = \frac{\frac{a^2}{2} A_c^2 P_m}{\frac{A_c^2}{2} + \frac{a^2}{2} A_c^2 P_m} = \frac{a^2 P_m}{1 + a^2 P_m}$$

Example

- The signal $m(t) = 3 \cos(200\pi t) + \sin(600\pi t)$ is used to modulate the carrier $c(t) = \cos(2 \times 10^{-5} t)$. The modulation index is $a=0.85$. Determine the power in the carrier component and in the sideband components of the modulated signal.

Demodulation of AM signals

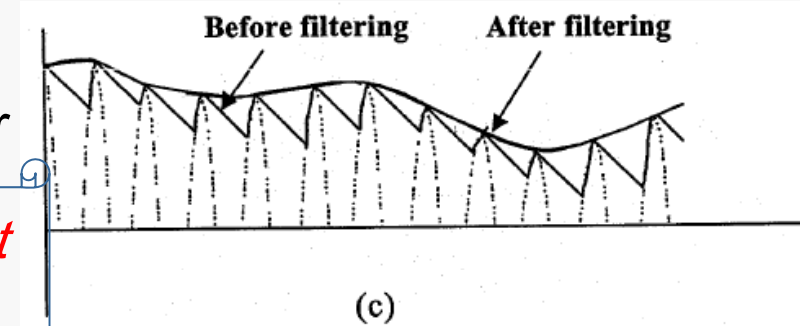
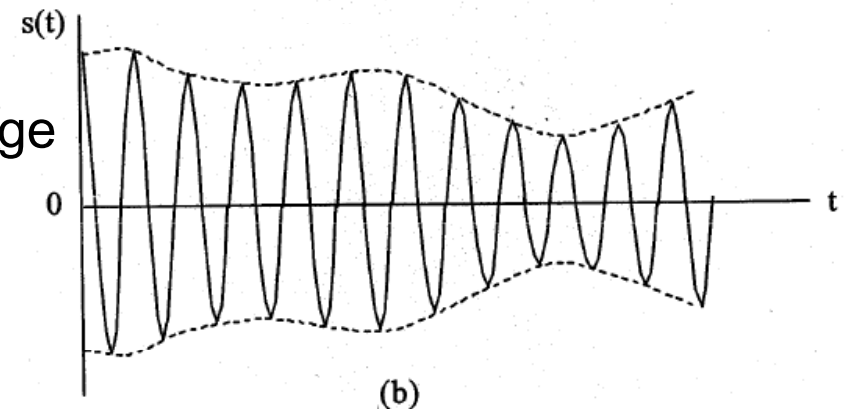
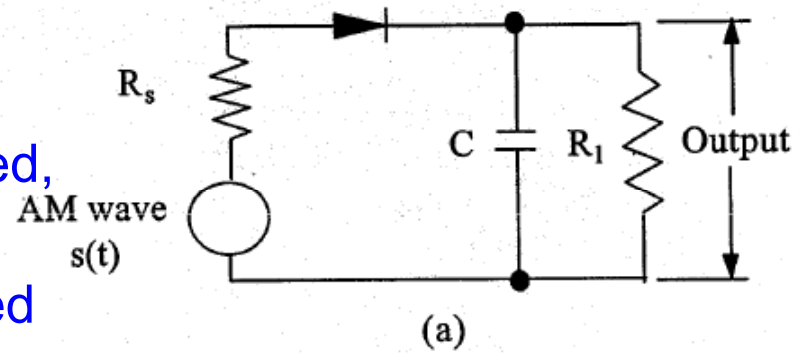
Envelope Detection

- On +ve half cycle, diode is forward-biased, capacitor is charged to peak value
- On -ve half cycle, diode is reverse-biased and capacitor discharges slowly through load resistor R_L .
- Assume AM wave was supplied by voltage source with internal impedance R_s .
- Also assume short charging time, i.e. $R_s C \ll 1/f_c$ and long discharging time, i.e.

$$1/f_c \ll R_L C \ll 1/W.$$

- Ripple can be removed by low-pass filter

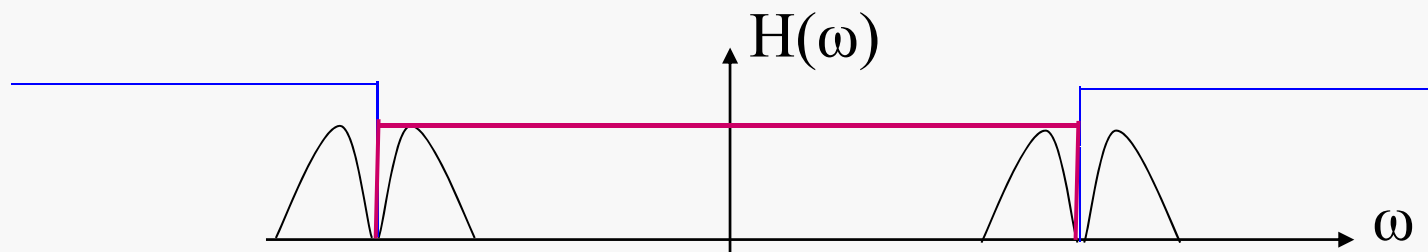
Envelope Detector is Simple and efficient when percentage modulation < 100%



Single Sideband (SSB) AM

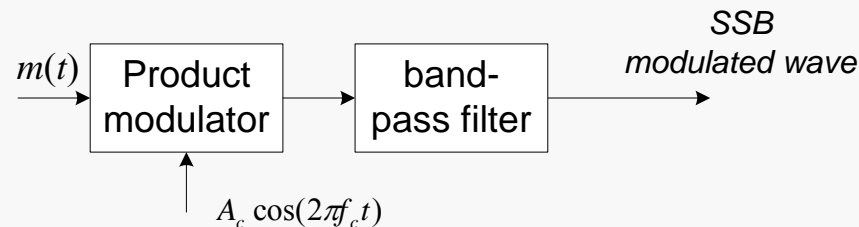
- Common problem in AM and DSBSC:
bandwidth wastage because the transmission bandwidth equals to twice the message bandwidth

⇒ SSB is very bandwidth efficient



Generation of SSB Waves: Frequency discrimination method

- ❑ Requirements on message signal $m(t)$
 - Little or no low-frequency components, i.e. “holes” at 0Hz. E.g: audio signal (speech or music). In telephony, the useful frequency content of a speech signal is restricted to 0.3 ~ 3.4 kHz
 - The highest frequency component $W \ll$ carrier frequency f_c
- ❑ Block diagram



- ❑ Two conditions for bandpass filter
 - Passband occupies the same frequency range as desired SSB wave
 - Guardband, separating the passband from the stopband where the unwanted sideband of the filter input lies, should be less than twice the lowest frequency, f_b in $m(t)$, i.e. must be between $f_c - f_b$ to $f_c + f_b$

Expression of SSB signals

- The baseband signal can be written as the sum of finite sinusoid signal

$$m(t) = \sum_{i=1}^n x_i \cos(2\pi f_i t + \theta_i), \quad f_i \leq f_c$$

- Then its USB component is

$$m_c(t) = \frac{A_c}{2} \sum_{i=1}^n x_i \cos[2\pi(f_c + f_i)t + \theta_i]$$

- After manipulation

$$\begin{aligned} m_c(t) &= \frac{A_c}{2} \left\{ \left[\sum_{i=1}^n x_i \cos(2\pi f_i t + \theta_i) \right] \cos 2\pi f_c t - \underbrace{\left[\sum_{i=1}^n x_i \sin(2\pi f_i t + \theta_i) \right]}_{\text{Hilbert transform of } m(t)} \sin 2\pi f_c t \right\} \\ &= \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} \widehat{m}(t) \sin 2\pi f_c t \end{aligned}$$

$$S_{SSB}(\omega) = \frac{1}{2}[M(\omega - \omega_c) + M(\omega + \omega_c)]H(\omega)$$

$$H(\omega) = \frac{1}{2}[Sgn(\omega + \omega_c) - Sgn(\omega - \omega_c)]$$

$$S_{SSB}(\omega) = \frac{1}{4}[M(\omega + \omega_c) + M(\omega - \omega_c)]$$

$$(\Leftrightarrow \frac{1}{2}m(t) \cos \omega_c t)$$

$$+ \frac{1}{4}[M(\omega + \omega_c)Sgn(\omega + \omega_c) - M(\omega - \omega_c)Sgn(\omega - \omega_c)]$$

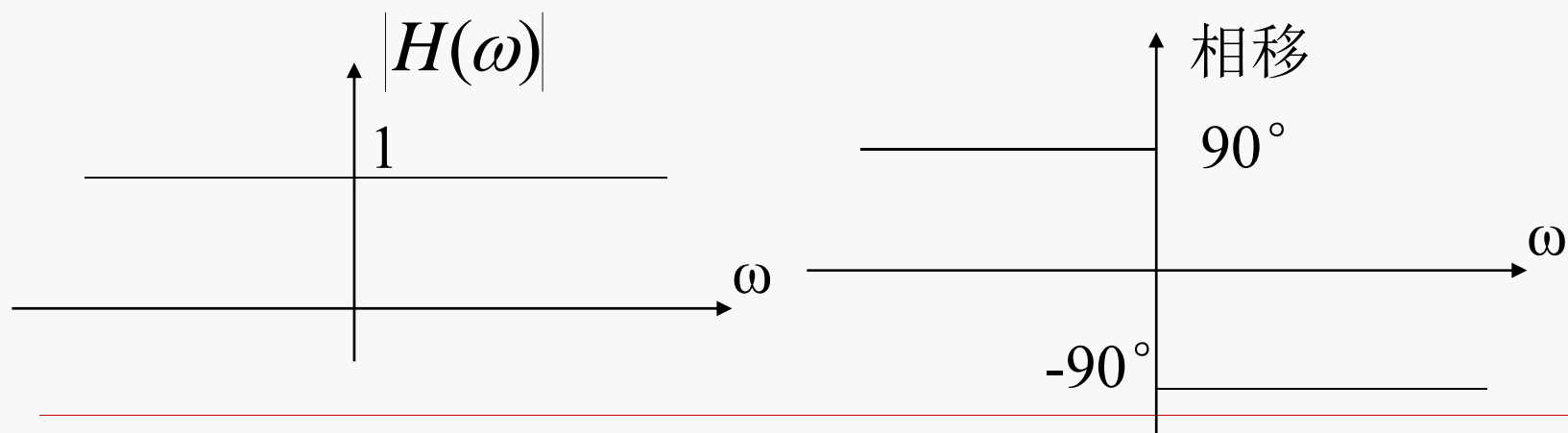
$$(\Leftrightarrow \frac{1}{2}\hat{m}(t) \sin \omega_c t)$$

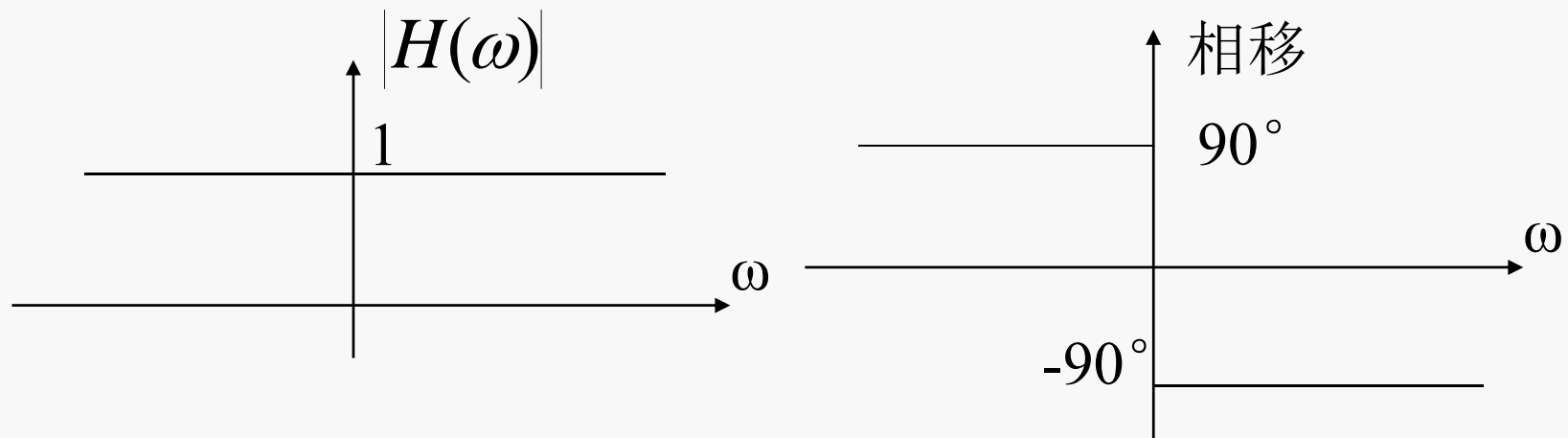
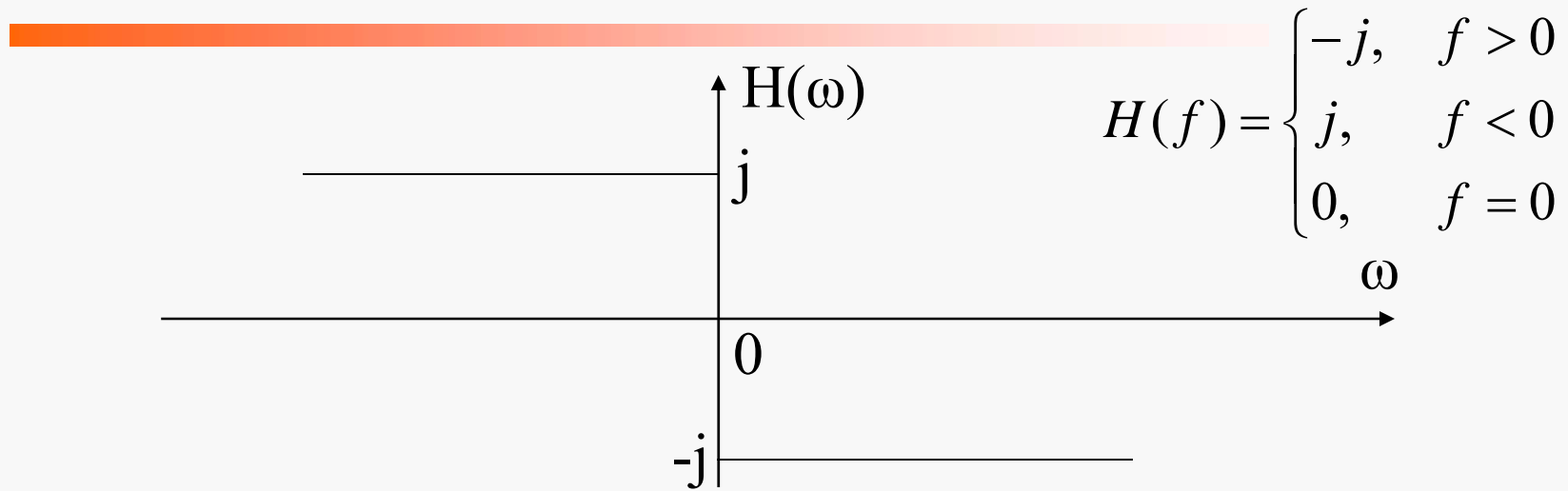
About Hilbert Transform

$$\square \quad x(t) \Leftrightarrow \hat{x}(t) \quad \Rightarrow \quad \hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau = x(t) * \frac{1}{\pi t}$$

$$X(\omega) \Leftrightarrow \hat{X}(\omega) \quad \Rightarrow \quad \hat{X}(\omega) \stackrel{\dot{}}{=} [-j \operatorname{sgn}(\omega)] X(\omega)$$

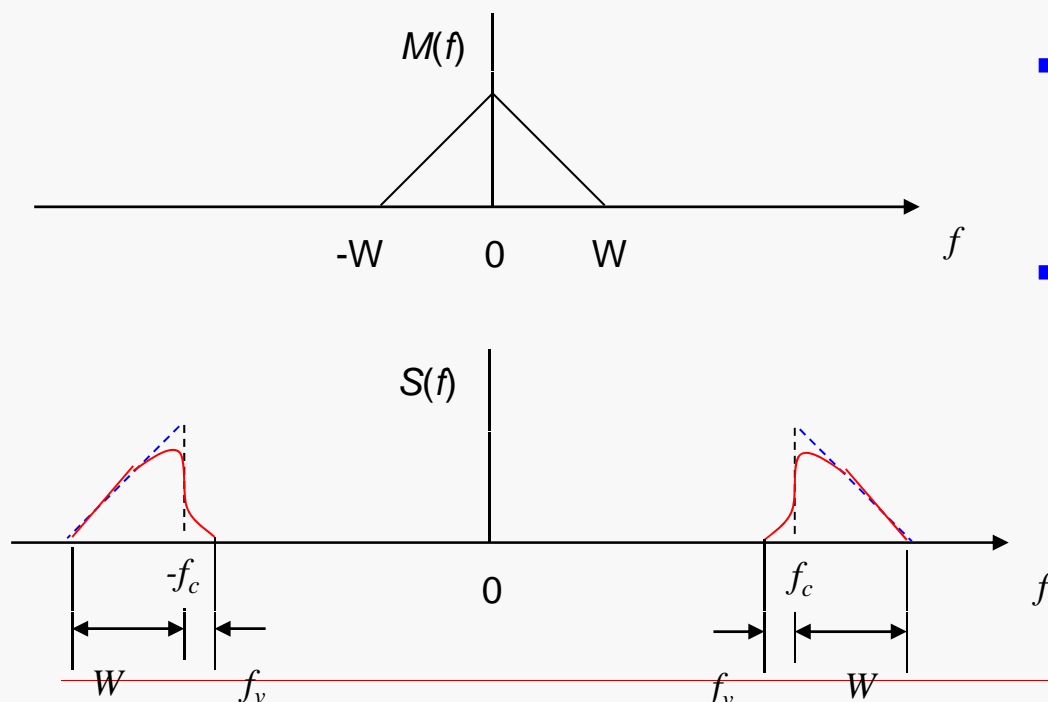
$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \\ 0, & f = 0 \end{cases}$$





Vestigial Sideband: VSB

- ❑ SSB is not suitable when signals have significant low frequency components
- ❑ **VSB** is a compromise between SSB and DSBSC
- ❑ VSB frequency domain description



- VSB signal bandwidth is $B = W + f_v$
 f_v : width of the vestigial sideband
- VSB is used in TV broadcasting and similar signals where good phase characteristics are required and low frequency components are significant

Comparison of AM Techniques

- ❑ **Conventional AM** demodulation uses simple envelop detector or square-law detector. Avoids complexity of coherent detection. E.g. **AM radio broadcast systems**
- ❑ **Suppressed-carrier** systems are more power efficient, making transmitters less expensive. Suitable for point-to-point transmissions
- ❑ **SSB modulation** requires minimum transmitter power and bandwidth. Suitable for point-to-point and over long distances
- ❑ **VSB** bandwidth requirements are between SSB and DSBSC. **Suitable for TV transmission**
- ❑ In **SSB and VSB**, the role of the quadrature component is to interfere with the in-phase component so as to eliminate power in one of the sideband achieve bandwidth saving

Frequency-Division Multiplexing

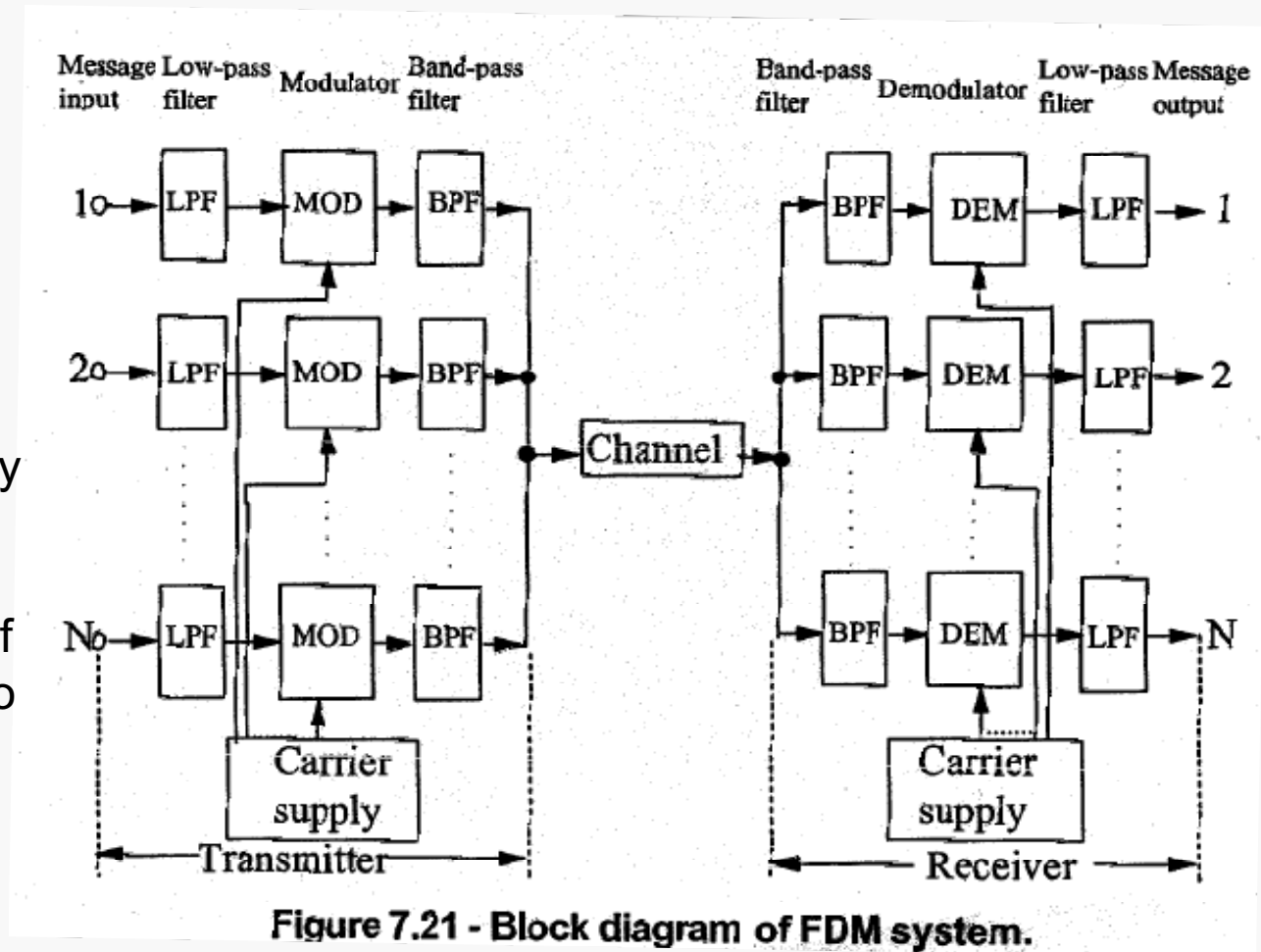
- ❑ Multiplexing is a technique where a number of independent signals are combined and transmitted in a common channel
- ❑ These signal are **de-multiplexed** at the receiver
- ❑ Two common methods for signal multiplexing
 - TDM (time-division multiplexing): usually used to transmit digital information
 - FDM (frequency-division multiplexing): may be used for either analog or digital signal transmission

Block Diagram of FDM

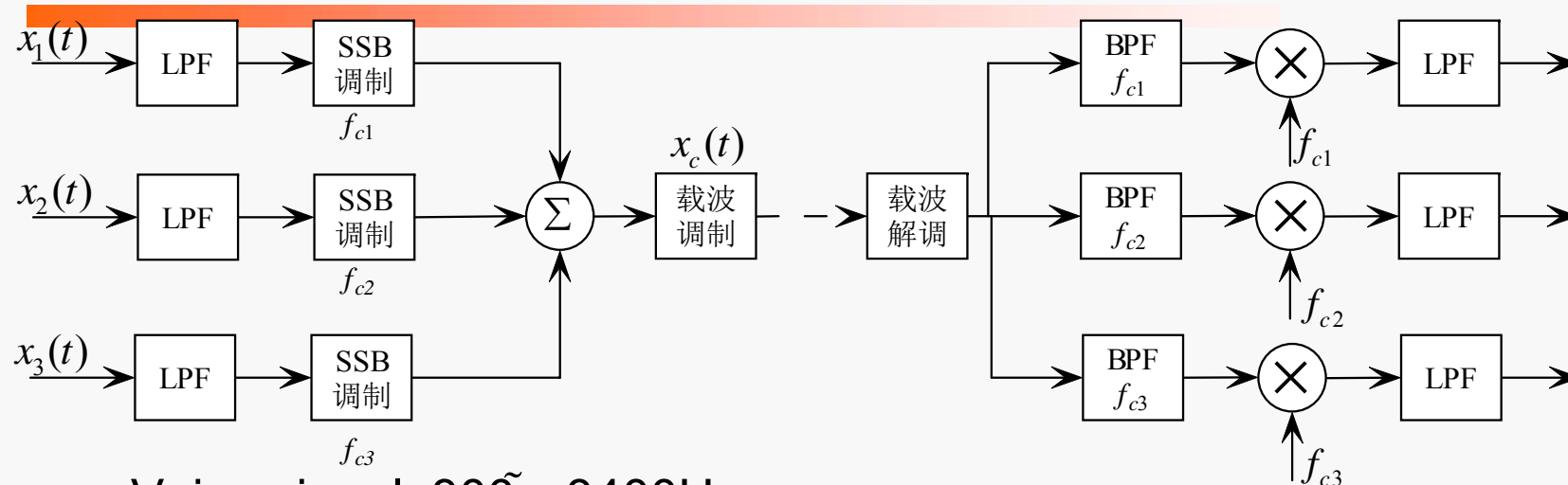
LPF: ensure signal bandwidth limited to W

MOD (modulator): shift message frequency range to mutually exclusive high frequency bands

BPF: restrict the band of each modulated wave to its prescribed range



FDM application in Telephone comm.



- ❑ Voice signal: 300 ~ 3400Hz
- ❑ Message is SSB modulated.
- ❑ In 1st-level multiplexing, 12 signals are stacked in frequency, with a freq. separation of 4 kHz between adjacent carriers
- ❑ A composite 48 kHz channel, called a **group channel**, transmits 12 voice-band signals
- ❑ Higher-order FDM is obtained by combining several group channels => **FDM hierarchy in telephone comm. systems**

Quadrature-Carrier Multiplexing

- **Quadrature-carrier multiplexing**: transmit two messages on the same carrier as

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

- $\cos()$ and $\sin()$ are two quadrature carriers
 - Each message signal is modulated by DSB-SC
- Demodulation of $m_1(t)$:

$$s(t) \cos(2\pi f_c t) = A_c m_1(t) \cos^2(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

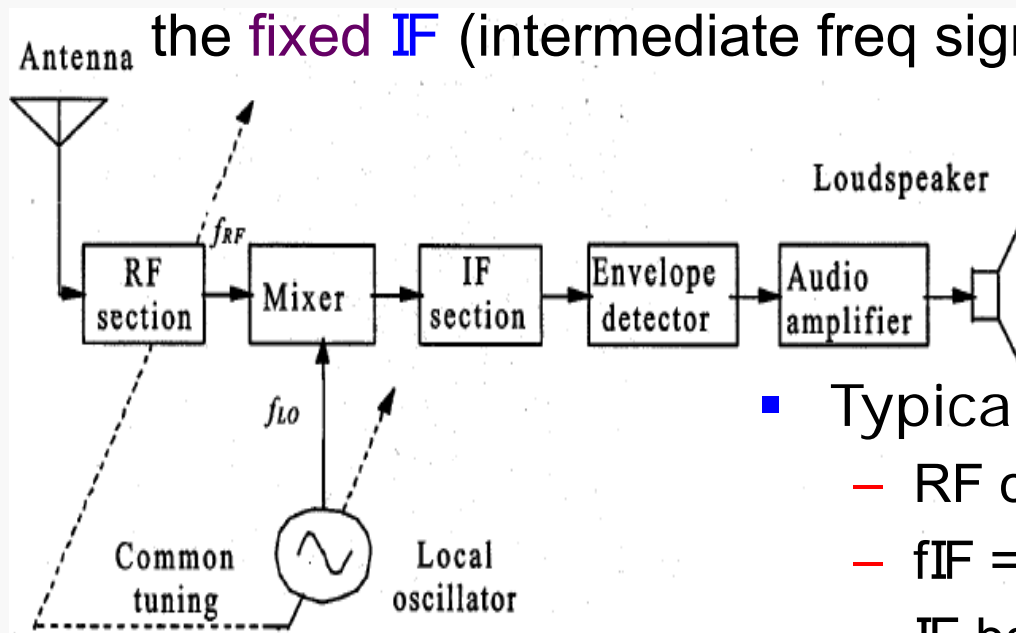
$$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t) \cos(4\pi f_c t) + \frac{A_c}{2} m_2(t) \sin(4\pi f_c t)$$



LPF

Application: AM Radio Broadcasting

- Commercial AM radio uses conventional AM
- The radio receiver is of the **superheterodyne** type, i.e. involves the freq conversion or heterodyning from the **variable** carrier freq of the incoming **RF** (radio freq) signal to the **fixed IF** (intermediate freq signal)



- Typical freq parameters
 - RF carrier range = 0.535 ~ 1.605 MHz
 - fIF = 455kHz
 - IF bandwidth = 10kHz